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INTENSIONAL LOGIC WITHOUT TEARS

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Introduction

W. V. Quine's "Methods of Logic" (Quine, 1974) is a goldmine of information for those seeking help with problems in formal logic. In chapter 29 (p 161) he outlines his "Main Method" of tackling problems of proof in first order predicate calculus. Quine is always at pains to eschew what he calls "monolithic" methods of proof, but the "Main Method" remains a very useful tool for demonstrating, by an almost mechanical procedure, that a set of premises is inconsistent. Logical truth may then be tested by showing the inconsistency of its negation, and it is not necessary to wait for the inspiration of a more elegant proof.

I feel sure that many linguists would like to have some such useful technique for grappling among the formulae of intensional logic. It seems to me that Quine's Main Method (I shall drop the inverted commas from now on) may be extended without too much difficulty to cope with such elaborate forms of logic, and it is the purpose of this article to demonstrate that this is so.

It is my belief that students of logic benefit most from a "hands on" approach to the subject – after trying things out oneself, and falling into all the traps for the unwary, it is much easier to appreciate the need for rigour, and easier to understand how such rigour is to be achieved. Now I must admit that I myself am, and no doubt always will be, just a student of logic, and as such prone to fall into traps. While I am convinced that the extension of the Main Method I present does work in intensional logic, I am not so confident that my presentation is flawless, that my language and metalanguage are kept properly distinct, and so on. I would be grateful for any criticisms from interested readers. As for the English language, I have not drawn any distinctions between "valid" and "logically true", or between "formula" and "sentence". Whether I should have done is one of those things I am not so sure of.

In a previous article (Stirk, 1982), I have already presented a simple introduction to the variation of the Main Method which is my starting point here, which I will not repeat. Rather, I will present an important example of the use of the method, together with a commentary to explain the procedure. Read in conjunction with Quine's presentation, this should provide sufficient introduction to the technique.

The Main Method in Higher Order Predicate Calculi

In the second order predicate calculus, it is possible to define the relation of identity in the following way (see, for example, Carnap, 1958, p 69):

$$(x)(y)(x = y \ . \equiv (P)(Px \equiv Py))$$

It will be seen that I am using the elegant combination of dot and bracket notations explained in Quine (1974). The problem is to show that the use of this definition is equivalent to the use of the following two axioms of identity (see Quine, 1974, p 221 et seq):

$$(x)(x = x)$$
$$(x)(y)(P)(x = y \,.\, Px \,.\supset Py)$$

The second, of course, is the well-known Leibniz' law. The first part of the proof shows that the definition formula does indeed follow from the two axioms:

1. $(x)(x = x)$
2. $(x)(y)(P)(x = y \,.\, Px \,.\supset Py)$
3. $(\exists x)(\exists y)(x \neq y \,.\, (P)(Px \equiv Py) \,.v.\, x = y \,.\, (\exists P)-(Px \equiv Py))$
4. $a \neq b \,.\, (P)(Pa \equiv Pb) \,.v.\, a = b \,.\, (\exists P)-(Pa \equiv Pb))$ 3

10.	$a \neq b$	
11.	$(P)(Pa \equiv Pb)$	
12.	$[(\lambda x)(a = x)]a \equiv [(\lambda x)(a = x)]b$	11
13.	$a = a \,.\equiv.\, a = b$	12
14.	$a = a$	1
15.	$a = b$	13, 14
16.	⊠	10, 15

20.	$a = b$	
21.	$(\exists P)-(Pa \equiv Pb)$	
22.	$-(Fa \equiv Fb)$	21
23.	$a = b \,.\, Fa \,.\supset Fb$	2
24.	$Fa \supset Fb$	20, 23
25.	$a = b \,.\, -Fa \,.\supset -Fb$	2
26.	$-Fa \supset -Fb$	20, 25
27.	$Fb \supset Fa$	26
28.	$Fa \equiv Fb$	24, 27
29.	⊠	22, 28

The premise in line 3 above is the negation of the definition formula, and the proof works by showing that this line is inconsistent with the axioms in lines 1 and 2. In line 4, the existential quantifiers of line 3 are instantiated with the letters "a" and "b", a "new" letter for the second instantiation in accordance with the rules of the Main Method. The number "3" to the right indicates that the line was derived from line 3. In line 4, none of the quantifiers are prenex. Quine (1974) deals with this situation by providing rules for moving quantifiers to prenex position (p 118 et seq). Rather than use these, which are sometimes tricky to apply, I have preferred to use a tableau technique, as outlined in, for instance, Hodges (1977). This technique is guaranteed to "split up" lines in such a way that only prenex quantifiers ever need to be instantiated. In this case, the main connective of line 4 is an alternation, so here the tableau divides into two branches, one to test each side of the alternation. The left hand side is a conjunction which is separated as lines 10 and 11. Similarly the right hand conjunction becomes two lines 20 and 21. In line 12 the universal quantifier "P" is instantiated, a quantifier which ranges over first order predicates. The rules for instantiation are the same: a universal quantifier of this kind may be instantiated with any first order predicate. The difficulty now is to instantiate it with a predicate which is most likely to give rise to an inconsistency. This sometimes requires some ingenuity. The predicate chosen in 12, "$(\lambda x)(a = x)$", is a suitable one — line 15 is reached, which is inconsistent with 10. The inconsistency is indicated by the sign "⊗", which is borrowed from Lewis Carroll, who pioneered the tableau method in work which unfortunately remained unpublished. (See Bartley, 1977).

The situation on the right hand branch is simpler. Here it is an existential quantifier in line 21 that needs instantiating. The neutral letter "F" is chosen, and once more an inconsistency is reached, completing the proof.

Proving that the definition formula implies the axioms is best done in two stages. First it is shown that "$(x)(x = x)$" can be derived:

1.	$(x)(y)(x = y \;.\equiv\; (P)(Px \equiv Py))$	
2.	$(\exists x)(x \neq x)$	
3.	$a \neq a$	2
4.	$a = a \;.\equiv\; (P)(Pa \equiv Pa)$	1
5.	$(\exists P)(Pa . -Pa \;.v.\; -Pa . Pa)$	3, 4
6.	$Fa . -Fa$	5
7.	$\otimes$	6

A simple matter. As for the derivation of Leibniz' law:

1.	$(x)(y)(x = y \;.\equiv\; (P)(Px \equiv Py))$	
2.	$(\exists x)(\exists y)(\exists P)(x = y \;.\; Px \;.\; -Py)$	
3.	$a = b \;.\; Fa \;.\; -Fb$	2
4.	$a = b \;.\equiv\; (P)(Pa \equiv Pb)$	1
5.	$(P)(Pa \equiv Pb)$	3, 4
6.	$Fa \equiv Fb$	5
7.	Fb	3, 6
8.	$\otimes$	3, 7

Also quite straightforward. It seems that the Main Method will apply in such higher order calculi. The next step is to apply it to the various systems of modal logic.

Modal Propositional Calculus

In this section I shall adopt the symbols "L" and "M" for the necessity and possibility operators respectively, following the example of Hughes and Cresswell in their excellent standard textbook on modal logic (1968). Otherwise I shall continue to use Quine's notation: brackets will be required with "L" and "M" as they are with quantifiers in that notation.

"L" may be read, "it is necessary that..." and "M", "it is possible that...". Bearing in mind the possible world interpretation, however, and with certain qualifications to be made clear later, they may be read, "for all possible worlds..." and "for at least one possible world..." respectively. It is this reading that gives the clue to a way of extending Quine's main method to cover modal logics too. Just as the existential quantifier required instantiation with a new individual name, so "M" may be taken to require "instantiation" with a "new world", whereas "L" can be instantiated with any world. An example or two will make this more clear.

Consider the premises:

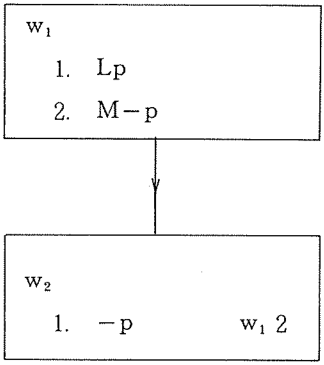
1. Lp
2. M−p

These make a good introductory example, since they are obviously inconsistent, bearing in mind that "Lp" is equivalent to "−M−p". As usual, the proof of inconsistency begins by assuming the premises to be true, this time in a particular world w_1. The world will be represented by a rectangle surrounding any sentences assumed or inferred to be true in it, with its name w_1 in the top left-hand corner:

w_1

1. Lp
2. M−p

Now the possibility operator in premise 2 is instantiated with a new world w_2 :



According to that second premise, there is at least one other world in which " −p " is true, and that is precisely what the diagram indicates. The significance of the arrow joining the two worlds will become clear later. As for notation, it seems convenient to start a new numbering in each world; references to other worlds in justifying a line may be made by using a world number as well as a line number, as has been done above.

The proof is now simple enough to complete. According to the first premise, " p " is true in every world, so it is true in w_2 as well as w_1 : this provides the expected inconsistency, as shown below in the completed proof:

w_1

1.	Lp	
2.	M−p	

↓

w_2

1.	−p	w_1 2
2.	p	w_1 1
3.	$\boxtimes$	1, 2

In the next few examples, I shall give first a complete symbolic proof, followed by any necessary commentary.

This, while simple, is quite instructive:

w_1

1.	M(p. −p)	

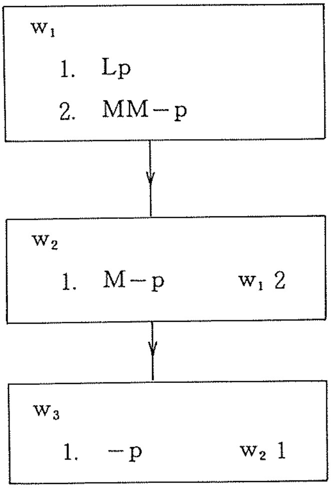
↓

w_2

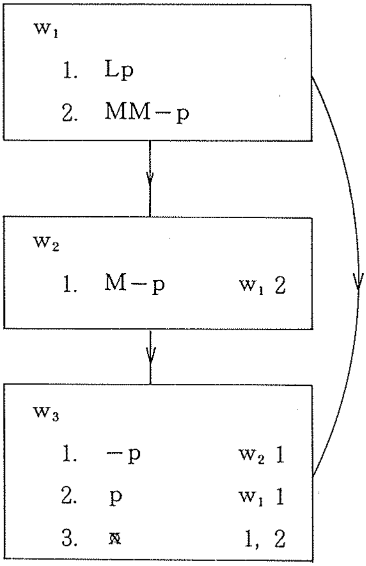
1.	p. −p	w_1 1
2.	$\boxtimes$	1

Since the single premise "M(p.−p)" turns out to be inconsistent, its negation, "L(p v −p)", must be logically true. "p v −p" is of course a tautology, and a moment's reflection shows that all tautologies are *necessarily* true in this logic, for a similar proof will hold in each case.

This next proof has particular importance:



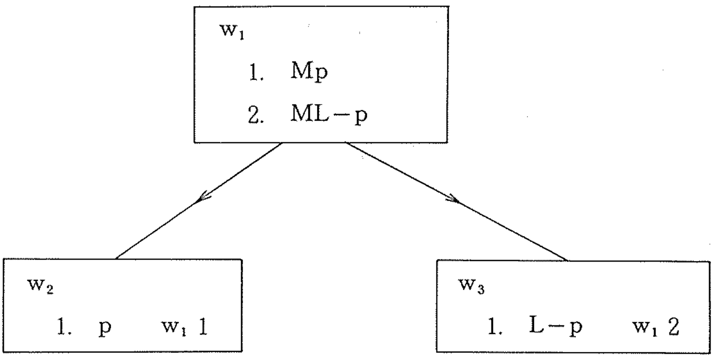
There would be an inconsistency in w_3 if we added a second line "2. p w_1 1". Whether this addition is justified or not depends on how we interpret the sentence "Lp". True, reading "L" as "in all possible worlds" suggests that "p" should be true in w_3 also, but there is also the possibility of interpreting "L" in such a way that only worlds directly connected to w_1, like w_2 in the diagram, are "accessible" to it. In that case, "L" is read as "in all *accessible* worlds", and the "Lp" in w_1 means only that "p" must be true in w_2, but not necessarily in w_3, in this example. The system of modal propositional calculus which arises from this interpretation is generally known as "system T". The example above shows that the premises "Lp" and "MM−p" can consistently both be true in T. The arrows in the diagram show the "accessibility relation" holding between worlds − w_2 is accessible from w_1, but w_3 is not. If w_3 were also accessible from w_1, we would have the following situation:



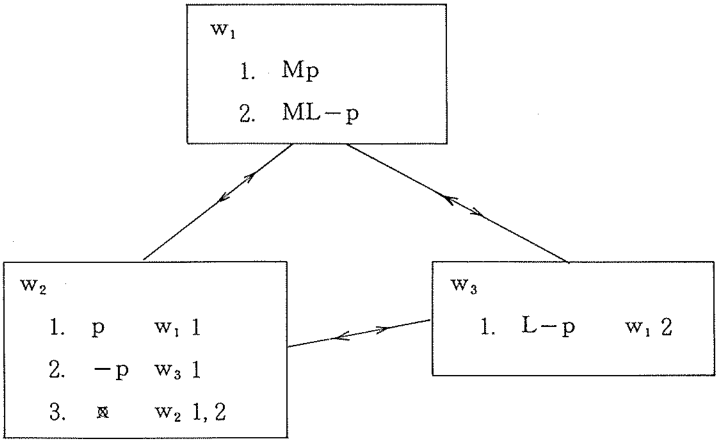
The accessibility of w_3 from w_1 can be assured by supposing the accessibility relation to be transitive. The two premises turn out inconsistent under this supposition, showing that "−(Lp. −MMp)", or "Lp ⊃ LLp", is logically true in this system, which usually has the label S4. More details about the various systems may be found in Hughes & Cresswell (1968). My purpose here is merely to show how the Main Method is easily extended to cover them.

The diagrams which result are closely related to the semantic tableaux given by Hughes and Cresswell, but it is important to bear in mind a vital difference. The Main Method proceeds by assuming that a set of premises is true, and showing that this leads to inconsistency —Hughes and Cresswell, following Kripke, assume that a formula is *false* and follow the trail from there to inconsistency. The results are the same as far as propositional calculi are concerned, but only the Main Method is suitable for predicate calculi. For the sake of consistency of approach, then, I use the Main Method procedure for modal propositional calculi also.

Some more examples. The formula "Mp ⊃ LMp" is not logically true in S4:

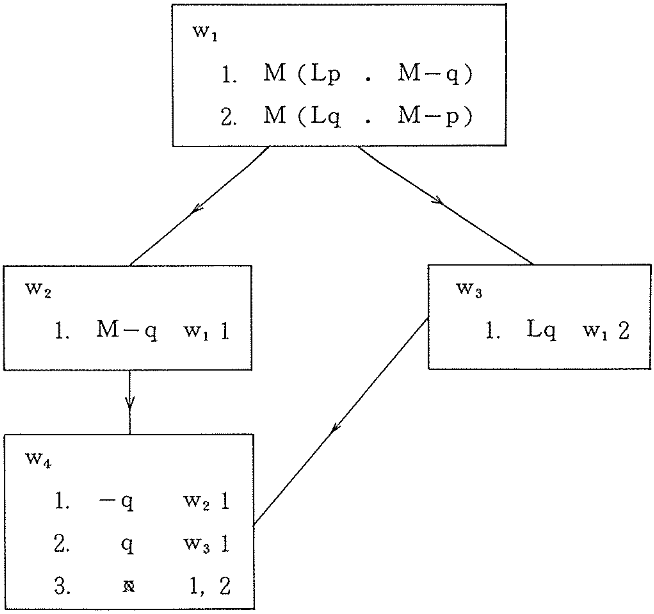


The transitivity of the accessibility relation does not help in bringing about any inconsistency here. If the relation is made reflexive, though, as well as transitive, then all worlds in a diagram will be made accessible to each other, and an inconsistency will be found:



Double arrows are used to show the reflexivity of this system, designated S5. This is the most important system for our purposes, since it figures in Montague (1970), as will be seen below.

The following examples demonstrate the logical truth of various more complicated sentences in the three systems, and illustrate certain helpful modifications to the basic Main Method. Firstly, it will be shown that "L (Lp ⊃ Lq) v L (Lq ⊃ Lp)" is logically true in S5:



Proofs involving more complex expressions can become overfull of worlds and festooned with arrows. It is possible to make useful simplifications by remembering that "Lp ⊃ LLp" is logically true in S4 and S5, and that "Mp ⊃ LMp" is logically true in S5. The previous proof looks simpler as:

w_1

1.	M (Lp . M−q)	
2.	M (Lq . M−p)	

↓

w_2

1.	Lp	w_1 1
2.	M (Lq . M−p)	w_1 2, S5

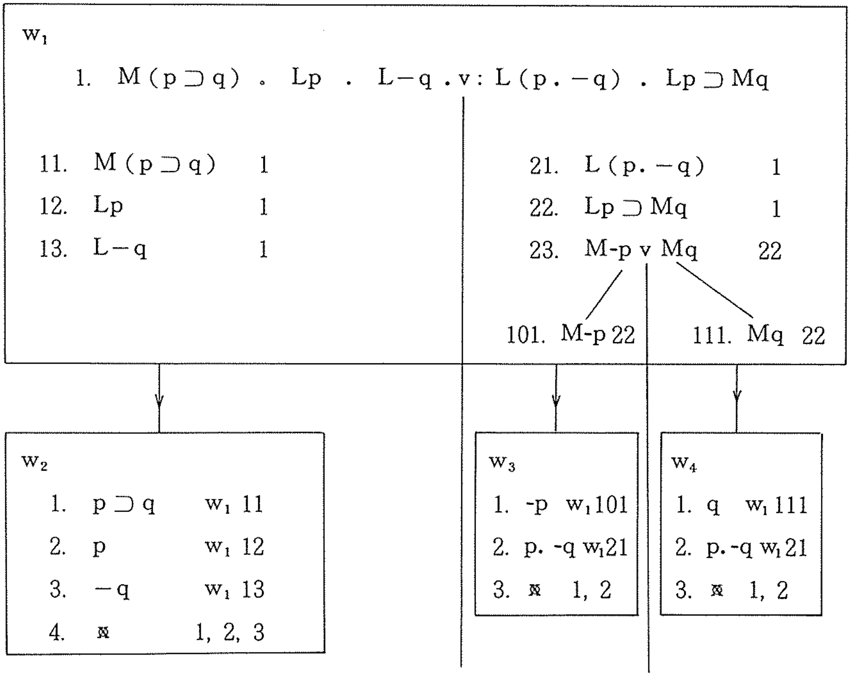
↓

w_3

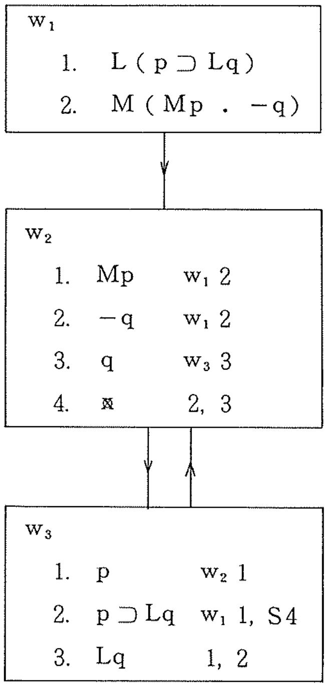
1.	M−p	w_2 2
2.	Lp	w_1 1, S4
3.	⊠	1, 2

Here I have indicated the use of "Lp ⊃ LLp" by writing "S4" to the right of the line that depends on this use; "S5" similarly shows a use of "Mp ⊃ LMp".

The next example shows how the tableau method for alternation may be adapted to modal calculi. It seems best here to draw a clear vertical line dividing the diagram in two after each alternation, for of course as usual each branch must be considered quite separately. This particular proof shows that "M (p ⊃ q) ≡ . Lp ⊃ Mq" is logically true in T:

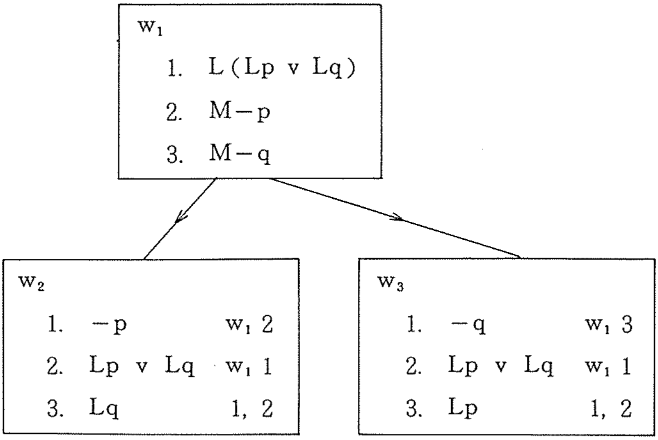


The following example shows that "L(p⊃Lq)⊃L(Mp⊃q)" is logically true in S5:

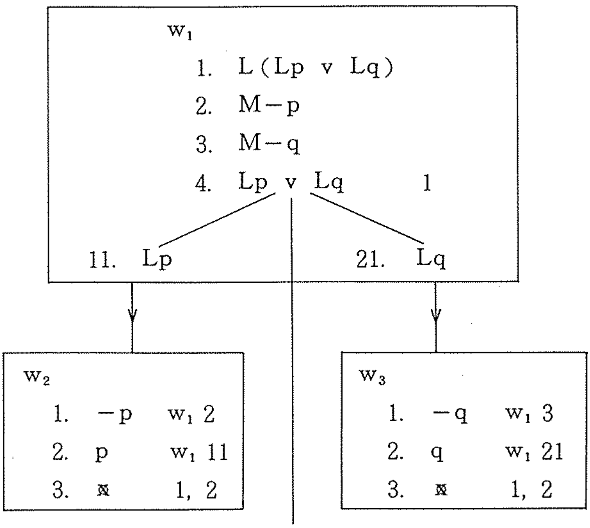


In order to shorten this proof, it was useful to remember the reflexive property of S5, and return from w_3 to w_2, writing "q" as the third line of w_2 as a consequence of w_3 3, instead of going on to further worlds.

In the first order predicate calculus the Main Method requires that existential quantifiers should be instantiated before universal ones whenever possible, in order to reach a speedier conclusion. An analagous rule, that "M" should be instantiated before "L", does not apply. This may be seen by showing that "L(Lp v Lq) ⊃ .Lp v Lq" is logically true in T. That is of course trivially the case, but a rigid application of the Main Method results in:



The inconsistency has been missed here. Instantiating w_1 1 first gives better results:



The Main Method as applied to modal propositional calculus does not work quite so mechanically as it does for first order predicate calculus.

Modal Predicate Calculi

Introducing modality into predicate calculi presents us with a number of arbitrary choices, whose consequences may be explored with a simply extended Main Method. How far should the populations of individuals in each world overlap? Should the universal quantifier range just over the population of a particular world, or over all possible individuals? As an example, let us explore some of the circumstances under which the well-known *Barcan formula*, "(x) LFx ⊃ L (x) Fx", is or is not logically true. The following situation will certainly arise:

w_1

1.	(x) LFx	
2.	M (∃x) −Fx	

↓

w_2

1.	(∃x) −Fx	w_1 2
2.	−Fa	1

The next step would be to go back to w_1 and instantiate the universal quantifier in line 1 with "a", yielding "LFa", which would result in an inconsistency in w_2 :

w_1

1.	(x) LFx	
2.	M (∃x) −Fx	
3.	LFa	1

↓

w_2

1.	(∃x) −Fx	w_1 2
2.	−Fa	1
3.	Fa	w_1 3
4.	⊠	2, 3

This proof will only go through if the individual denoted by "a" is in the population of w_1 as well as w_2. In the simplest case, we could specify that every world has the same population, so that the Barcan formula would be logically true whether the modal system is T, S4 or S5.

A more complicated possibility is to suppose that the population of w_2 is larger than that of w_1, so that "a", introduced in line w_2 2, does not figure in w_1. As a result, line $w_1$3 is no longer justified, if the universal quantifier in line w_1 1 is taken as referring only to the population of w_1. No inconsistency arises now, so the Barcan formula is not logically true in this system. This is so whether it is T or S4 that is under consideration. The condition is, roughly speaking, that if world j is accessible from world i, then the population of j should be larger than that of i. This condition, known as the *inclusion requirement*, cannot hold in S5, where the accessibility relation is reflexive, so under this interpretation the Barcan formula must be logically true when lower predicate calculus is added to S5.

A difficulty that arises when worlds have different populations is that of the status of, say, the sentence "Fa" in a world where "a" is not a member of the population. The most austere logical view is to demand that sentences must always be either true or false, and Kripke has shown that it is possible to build a model of modal predicate calculus on this basis. In this model, different worlds may have quite different populations, so the Barcan formula is not logically true even in S5. Much more surprising is that even a formula like "$(x)Fx \supset Fy$", where "y" refers to some individual, is not logically true in Kripke semantics. This is because "Fy" may be false in some world where "y" is not a member of the population —a world where "$(x)Fx$" is nevertheless true.

One way around this apparently paradoxical result, at least in cases where the inclusion requirement holds, is to maintain that sentences like "Fy" above are neither true nor false but *undefined* in worlds not containing "y". A thorough investigation of this notion is to be found in Hughes and Cresswell (1968), p 170 et seq. I will abandon further discussion of it here, since Montague seems to favour Kripke semantics in general, and in his (1970) employs a simple system where each world has the same population of "possible individuals". It should be apparent, though, that the Main Method may be used to explore any of these models.

Even in the very simplest model, a curious result may arise which deserves to be known as "Quine's Paradox". A particularly succinct statement and discussion of it is to be found in Ayer (1973), p 196 et seq. The paradox is that things that are identical are necessarily identical, or, symbolically, that "$(x)(y)[x=y . \supset L(x=y)]$" is logically true. The Main Method demonstration runs as follows:

w_1

1.	$(\exists x)(\exists y)[x = y . M(x \neq y)]$	
2.	$(x)(y)(P)(x = y . Px . \supset Py)$	Axiom
3.	$a = b$	1
4.	$M(a \neq b)$	1
5.	$(P)(a = b . Pa . \supset Pb)$	2
6.	$a=b . [(\lambda x)L(a=x)]a . \supset [(\lambda x)L(a=x)]b$	5
7.	$a=b . L(a=a) . \supset L(a=b)$	6
8.	$(x)L(x=x)$	Theorem
9.	$L(a=a)$	8
10.	$L(a=b)$	3, 7, 9

↓

w_2

1.	$a \neq b$	w_1 4
2.	$a = b$	w_1 10
3.	⊠	1, 2

The proof is quite straightforward, although it will be noticed that I have introduced an unproved theorem in line w_1 8, instead of the simple axiom of identity "(x)(x = x)". Of course axioms must be taken as true in every world, so it is permissible to assert the logical truth of "L(x)(x = x)", and I leave the proof of the formula "L(x)(x = x) ⊃ (x)L(x = x)", justifying the theorem, as an exercise for the interested reader. It is quite a simple exercise – unlike so many writers, I am genuinely trying to save space for more important matters, and not just avoiding the discussion of something horribly difficult.

The existence of Quine's paradox means that the usual axioms, or the definition, of identity cannot simply be imported into a modal predicate calculus. In the next section it will be seen how Montague deals with the difficulty in constructing his intensional logic.

Intensional Logic

We are now near the goal of using the Main Method as a tool for tackling intensional logic. I shall use the version presented in Montague (1970), since it is the one most familiar to linguists. Montague's notation is hardly ideal, and it differs markedly from the one I have used up to now, but nevertheless I shall use it for convenience of reference to Montague's original work.

The major difficulty we face with this logic is that it is not axiomatized. I do not know if anyone has yet found a suitable set of axioms for this version of intensional logic. What Montague provides is a recursive definition of the concepts of intension and extension (1970, pp 258−9). Instead of being able to set down straightforward axioms and definitions, as with, say, identity above, it is necessary to formulate statements and definitions which can be justified by referring to Montague's rules. The ways of doing this will, I hope, become clear as we proceed.

Montague's original recursive definition contained an error, fortunately not fatal, which is corrected by a footnote under Thomason's editorship (1970, p 259 footnote 10). Also I wish to simplify matters slightly for the logic by omitting any reference to time, that is Montague's set of moments J and the partial ordering $\leq$. This makes it convenient to repeat Montague's definition here, incorporating Thomason's correction and making the obvious simplifications:

(1) If α is a constant, then $\alpha^{\mathscr{I},i,g}$ is $F(\alpha)(i)$.

(2) If α is a variable, then $\alpha^{\mathscr{I},i,g}$ is $g(\alpha)$.

(3) If $\alpha \varepsilon ME_a$ and u is a variable of type b, then $[\lambda u\alpha]^{\mathscr{I},i,g}$ is that function h with domain $D_{b,A,I}$ such that whenever x is in that domain, $h(x)$ is $\alpha^{\mathscr{I},i,g'}$, where g' is the $\mathscr{I}$−assignment like g except for the possible difference that g' (u) is x.

(4) If $\alpha \varepsilon ME_{<a,b>}$ and $\beta \varepsilon ME_a$, then $[\alpha(\beta)]^{\mathscr{I},i,g}$ is $\alpha^{\mathscr{I},i,g}(\beta^{\mathscr{I},i,g})$ (that is, the value of the function $\alpha^{\mathscr{I},i,g}$ for the argument $\beta^{\mathscr{I},i,g}$).

(5) If $\alpha, \beta \varepsilon ME_a$, then $[\alpha = \beta]^{\mathscr{I},i,g}$ is 1 if and only if $\alpha^{\mathscr{I},i,g}$ is $\beta^{\mathscr{I},i,g}$.

(6) If $\varnothing \varepsilon ME_t$, then $[\neg\varnothing]^{\mathscr{I},i,g}$ is 1 if and only if $\varnothing$

is O; and similarly for $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$.

(7) If $\phi \in ME_t$ and u is a variable of type a, then $[\vee u\phi]^{\mathscr{I},i,g}$ is 1 if and only if there exists $x \in D_{a,A,I}$ such that $\phi^{\mathscr{I},i,g'}$ is 1, where g' is as in (3); and similarly for $\wedge u\phi$.

(8) If $\phi \in ME_t$, then $[\Box\phi]^{\mathscr{I},i,g}$ is 1 if and only if $\phi^{\mathscr{I},i',g}$ is 1 for all $i' \in I$, and similarly for $\Diamond\phi$.

(9) If $\alpha \in ME_a$ then $[\hat{}\alpha]^{\mathscr{I},i,g}$ is that function h with domain I such that whenever $i \in I$, then $h(i) = \alpha^{\mathscr{I},i,g}$.

It will be noticed that in (8) I have included a clause dealing with the possibility operator "$\Diamond$", which Montague does not employ in (1970), but which is indispensable for us. As well as that, it is necessary to introduce items to serve for instantiating variables in the Main Method. Using u and v as variables of type e, and x and y as variables of type $<s, e>$, as Montague does on page 260 of his (1970), k and l will be used as possible values of g(x), and a and b as possible values of g(u). Thus k and l are members of $D_{<s,e>,A,I}$ and a and b are members of $D_{e,A,I}$.

As a simple initial example I will show that "$\Box\wedge u[u={}^{\vee}{}^{\wedge}u]$" is logically true:

w_1	
1.	$\Diamond \bigvee u [u \neq {}^{\vee\wedge}u]$

↓

w_2		
1.	$\bigvee u [u \neq {}^{\vee\wedge}u]$	w_1 1
2.	$a \neq {}^{\vee\wedge}a$	1
3.	${}^{\wedge}a = \{ \langle w_1, a \rangle , \langle w_2, a \rangle \}$	Defn
4.	${}^{\vee\wedge}a = a$	3, Defn
5.	$\boxtimes$	2, 4

That seems to work in a fairly straightforward way. "Defn" at the end of a line means of course that Montague's definition has been appealed to. In the case of line w_2 3 here, we remember that "a" denotes a member of the set A of possible individuals, and has the same extension in each possible world. The intension is exhibited in line $w_2$3: it is the function h mentioned in clause 9. Line w_2 4 follows from w_2 3 and clause 10 of Montague's definition.

It should on the other hand be the case that "$\Diamond \bigvee x [x \neq {}^{\wedge\vee}x]$" is consistent in the logic. This means that we should *not* find a contradiction when we try:

w_1

1. $\Diamond \bigvee x [x \neq {}^\wedge{}^\vee x]$

↓

w_2

1.	$\bigvee x [x \neq {}^\wedge{}^\vee x]$	w_1 1
2.	$k \neq {}^\wedge{}^\vee k$	1
	Let k be $\{ \langle w_1 , a \rangle , \langle w_2 , b \rangle \}$	
3.	${}^\vee k = b$	Defn
4.	${}^\wedge{}^\vee k = {}^\wedge b = \{ \langle w_1 , b \rangle , \langle w_2 , b \rangle \}$	3, Defn

Here we seem to need a metalinguistic statement, the one that begins "Let..". This defines what we want "k" to be. It is a member of $D_{\langle s,e\rangle,A,I}$, and its definition is in accordance with that. The other uses of Montague's definition are similar to those in the previous proof; it will be seen that no inconsistency is reached.

Once some demonstrations of this kind have been made, it is possible to use the Main Method in a more "normal" way. Here is a more substantial example. On page 265 of his (1970), Montague claims that the formula "$\Box [\delta (x) \leftrightarrow \delta_* ({}^\vee x)]$" is a consequence of his meaning postulate (3) on page 263. It is, interestingly, actually equivalent to the meaning postulate. The first proof shows that the postulate implies the formula:

w_1

1. $\bigvee M \bigwedge x \Box [\, \delta(x) \leftrightarrow M\{{}^{\vee}x\}\,]$
2. $\bigvee x \Diamond [\, \delta(x) \wedge \neg \delta_*({}^{\vee}x) \;.\, v \,.\; \neg \delta(x) \wedge \delta_*({}^{\vee}x)\,]$
3. $\Diamond [\, \delta(k) \wedge \neg \delta_*({}^{\vee}k) \;.\, v \,.\; \neg \delta(k) \wedge \delta_*({}^{\vee}k)\,]$ 2
4. $\bigwedge x \Box [\, \delta(x) \leftrightarrow N\{{}^{\vee}x\}\,]$ 1

w_2

1. $\delta(k) \wedge \neg \delta_*({}^{\vee}k) \;.\, v \,.\; \neg \delta(k) \wedge \delta_*({}^{\vee}k)$ w_1 3

10. $\delta(k)$
11. $\neg \delta_*({}^{\vee}k)$
12. $\delta(k) \leftrightarrow N\{{}^{\vee}k\}$ w_1 4
13. $N\{{}^{\vee}k\}$ 10, 12
14. $\neg \delta({}^{\wedge\vee}k)$ 11
15. $\delta({}^{\wedge\vee}k) \leftrightarrow N\{{}^{\vee\wedge\vee}k\}$ w_1 4
16. $\bigwedge x [\, x = {}^{\vee\wedge}x\,]$ Th.
17. $\delta({}^{\wedge\vee}k) \leftrightarrow N\{{}^{\vee}k\}$ 15, 16
18. $\neg N\{{}^{\vee}k\}$ 14, 17
19. $\boxtimes$ 13, 18

20. $\neg \delta(k)$
21. $\delta_*({}^{\vee}k)$

etc.

The various steps in this proof should be clear enough. I have taken the liberty of introducing dots into Montague's notation, to avoid having to count brackets. Line w_2 14 follows from line w_2 11 because of the definition of "δ_*" given by Montague on page 265 of his (1970). Line w_2 16 is labelled "Th." for "theorem": it was demonstrated above that "$\Box \bigwedge u [u = {}^{\vee\wedge}u]$" is logically true, and it should be clear that the same proof would go through for variables of any type, according to the recursive definition. In the right hand branch I have written only "etc" after line 21, since, with appropriate changes of sign, it is similar to the left hand side. "N" represents an instance of the variable "M", discussed by Montague (1970, p 260).

The equivalence is established by showing that the implication holds in the opposite direction:

w_1

1.	$\bigwedge x \Box [\, \delta(x) \leftrightarrow \delta_*({}^\vee x) \,]$	
2.	$\bigwedge M \bigvee x \Diamond [\, \delta(x) \wedge \neg M\{{}^\vee x\} \ .v.\ \neg \delta(x) \wedge M\{{}^\vee x\}]$	
3.	$\bigvee x \Diamond [\, \delta(x) \wedge \neg {}^\wedge\delta_*\{{}^\vee x\} \ .v.\ \neg \delta(x) \wedge {}^\wedge\delta_*\{{}^\vee x\}]$	2
4.	$\Diamond [\, \delta(k) \wedge \neg \delta_*({}^\vee k) \ .v.\ \neg \delta(k) \wedge \delta_*({}^\vee k)\,]$	3

w_2

1.	$\delta(k) \wedge \neg \delta_*({}^\vee k) \ .v.\ \neg \delta(k) \wedge \delta_*({}^\vee k)$	w_1 4

10.	$\delta(k)$		20.	$\neg \delta(k)$
11.	$\neg \delta_*({}^\vee k)$		21.	$\delta_*({}^\vee k)$
12.	$\delta(k) \leftrightarrow \delta_*({}^\vee k)$	w_1 1		etc.
13.	$\otimes$	10, 11, 12		

Once again, when this proof branches, the right hand side is similar to the left. The only tricky point in this one is instantiating the "$\wedge M$" in line w_1 2. Only the choice of "$\hat{}\delta_*$" will do.

At first glance, Montague (1970) p265 seems to suggest that meaning postulate (2) also implies "$\Box[\delta(x) \leftrightarrow \delta_*({}^\vee x)]$". This is not the case, however, as the following demonstration suggests:

w_1

1.	$\bigwedge x \Box [\delta(x) \rightarrow \bigvee u \ x = \hat{}u]$	
2.	$\bigvee x \Diamond [\delta(x) \wedge \neg \delta_*({}^\vee x) \ .v. \ \neg \delta(x) \wedge \delta_*({}^\vee x)]$	
3.	$\Diamond [\delta(k) \wedge \neg \delta_*({}^\vee k) \ .v. \ \neg \delta(k) \wedge \delta_*({}^\vee k)]$	2

↓

w_2

1.	$\delta(k) \wedge \neg \delta_*({}^\vee k) \ .v. \ \neg \delta(k) \wedge \delta_*({}^\vee k)$	w_1 3

10.	$\delta(k)$		20.	$\neg \delta(k)$	
11.	$\neg \delta_*({}^\vee k)$		21.	$\delta_*({}^\vee k)$	
12.	$\delta(k) \rightarrow \bigvee u \ k = \hat{}u$	w_1 1	22.	$\delta(\hat{}{}^\vee k)$	20
13.	$\bigvee u \ k = \hat{}u$	10, 12			
14.	$k = \hat{}a$	13			
15.	$\delta(\hat{}a)$	10, 14			
16.	$\delta_*(a)$	15			
17.	$\neg \delta_*(a)$	11, 14			
18.	$\times$	16, 17			

In the course of this proof, I have tacitly used the definition of

" δ_* " in moving from line $w_2 15$ to $w_2 16$, and elsewhere. The derivation of line $w_2 17$ uses the fact, proved above, that " $\wedge x \Box [x = {}^{\vee}{}^{\wedge}x]$ ". It is also assumed that line $w_2 14$ sanctions the substitution of " $^{\wedge}a$ " for " k ", something that strictly speaking requires a proof based on the definitions.

Nevertheless, it is clear that although an inconsistency is revealed on the left hand side, there is no way of reaching one on the right, for the simple reason that " $^{\wedge\vee}k$ " may not be the same as " k ". All that can be proved, it seems, is that postulate (2) implies " $\wedge x \Box [\delta(x) \rightarrow \delta_*({}^{\vee}x)]$ ", a weaker statement than the one Montague presents.

Interestingly, though, this weaker formula seems to be sufficient, at least in deriving Montague's simpler examples (1970, p266 et seq). Although we cannot state " $\wedge x \Box [\delta_*({}^{\vee}x) \rightarrow \delta(x)]$ ", it is possible to write " $\wedge u \Box [\delta_*(u) \rightarrow \delta({}^{\wedge}u)]$ ", which follows directly from the definition of " δ_* ". The following examples show that this, together with the weak formula, is enough to demonstrate the equivalence of " $\vee x [\text{man}'(x) \wedge \text{walk}'(x)]$ " and " $\vee u [\text{man}'_*(u) \wedge \text{walk}'_*(u)]$ ".

1.	$\vee x [\text{man}'(x) \wedge \text{walk}'(x)]$	
2.	$\wedge u [\neg \text{man}'_*(u) \vee \neg \text{walk}'_*(u)]$	
3.	$\wedge x \Box [\text{man}'(x) \rightarrow \text{man}'_*({}^{\vee}x)]$	Theorem
4.	$\wedge x \Box [\text{walk}'(x) \rightarrow \text{walk}'_*({}^{\vee}x)]$	Theorem
5.	$\text{man}'(k) \wedge \text{walk}'(k)$	1
6.	$\text{man}'(k) \rightarrow \text{man}'_*({}^{\vee}k)$	3
7.	$\text{walk}'(k) \rightarrow \text{walk}'_*({}^{\vee}k)$	4
8.	$\text{man}'_*({}^{\vee}k) \wedge \text{walk}'_*({}^{\vee}k)$	5, 6, 7

9.	$\neg$ man'$_*$ ($^\vee$k) v $\neg$ walk'$_*$ ($^\vee$k)	2
10.	☒	8, 9

There is no need to draw a rectangle round this, since all the action takes place in just one world. In deriving line 9, the variable "u" was instantiated by "$^\vee$k": I suppose this is not strictly according to the rules, but it only suppresses an obvious intermediate step of stating that "$^\vee$k" is, say, "a".

The proof continues, using the definition of "δ_*" only:

1.	$\bigvee$u [man'$_*$ (u) $\wedge$ walk'$_*$ (u)]	
2.	$\bigwedge$x [$\neg$ man' (x) v $\neg$ walk' (x)]	
3.	$\bigwedge$u $\Box$ [man'$_*$ (u) $\rightarrow$ man' ($^\wedge$u)]	Defn. of δ_*
4.	$\bigwedge$u $\Box$ [walk'$_*$ (u) $\rightarrow$ walk' ($^\wedge$u)]	Defn. of δ_*
5.	man'$_*$ (a) $\wedge$ walk'$_*$ (a)	1
6.	man'$_*$ (a) $\rightarrow$ man' ($^\wedge$a)	3
7.	walk'$_*$ (a) $\rightarrow$ walk' ($^\wedge$a)	4
8.	man' ($^\wedge$a) $\wedge$ walk' ($^\wedge$a)	5, 6, 7
9.	$\neg$ man' ($^\wedge$a) v $\neg$ walk' ($^\wedge$a)	2
10.	☒	8, 9

Once again the proof is compressed by instantiating "x" directly with "$^\wedge$a", to produce line 9.

Conclusion

I hope that the examples given have adequately shown the power and value of the extended Main Method in the field of intensional logic. Without doubt it could be extended further to deal with moments of time, and the operators " W " and " H ", if a suitable notation were devised. Many textbooks use some system of " natural deduction " in elementary logic, which in some ways is similar to the Main Method. But I have never seen natural deduction employed in intensional logic, and I feel it would be far too involved and unwieldy to cope with the complexities. The Main Method sometimes needs some ingenuity in application, but nothing out of the ordinary. It is at the very least a handy compass when one's intuitions are all at sea.

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