



Title	Prediction of Welding Residual Stress, Deformation and Ultimate Strength of Plate Panels(Mechanics, Strength & Structural Design)
Author(s)	Ueda, Yukio; Yao, Tetsuya; Nakacho, Keiji et al.
Citation	Transactions of JWRI. 1992, 21(2), p. 277-283
Version Type	VoR
URL	<a href="https://doi.org/10.18910/9940">https://doi.org/10.18910/9940</a>
rights	
Note	

*The University of Osaka Institutional Knowledge Archive : OUKA*

<https://ir.library.osaka-u.ac.jp/>

The University of Osaka

# Prediction of Welding Residual Stress, Deformation and Ultimate Strength of Plate Panels †

Yukio UEDA\*, Tetsuya YAO\*\*, Keiji NAKACHO\*\*\* and M.G. YUAN\*\*\*\*

## Abstract

This paper describes an accurate and simplified method of estimation of compressive ultimate strength of a long rectangular plate panel in a longitudinally and transversely stiffened plate fabricated by welding. The procedure of estimation consists of three steps: (1)welding residual stress (2)welding deformation and (3)compressive ultimate strength with consideration of the effects of (2) and (3). The first two are estimated consistently in terms of inherent strain and deformation. A equivalent plate is used for (3), in which a complex initial deflection is idealized in a single deflection mode and formulae are derived for the ultimate strength of the equivalent plate.

**KEY WORDS :**(Prediction Method) (Compressive Ultimate Strength) (Stiffened Panel) (Welding Initial Deflection) (Welding Residual Stress) (Inherent Strain) (Inherent Deformation)

## 1. Introduction

In the design of plated structures, it is fundamentally required to estimate accurately the compressive strength of rectangular plates, surrounded by stiffeners or girders, under uni-axial load. The stiffeners and girders are attached usually by welding which produces residual stress and deformation in both plates and, stiffeners and girders.

In order to estimate the compressive strength of a rectangular plate with welding residual stresses and deformation the following steps of estimation should be taken,

- (1) welding residual stress
- (2) welding deformation
- (3) compressive strength under the influencing factors, such as welding residual stresses and deformation.

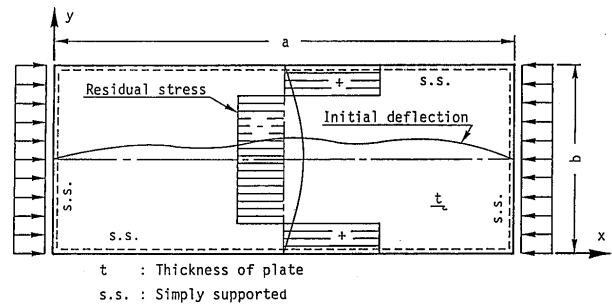
For estimation of the first two factors, (1) and (2), inherent strain and deformation are used as parameters, which are determined mainly by the condition of welding, the kind of steel, and affected little by stiffener space. Actual calculation may be carried out consistently by the finite element method on the above three factors, which the authors have applied.

However, in this report, based on results of numerical calculation by FEM, simple methods for estimating the first two items, (1) and (2), and the third one, (3), with formulae will be presented.

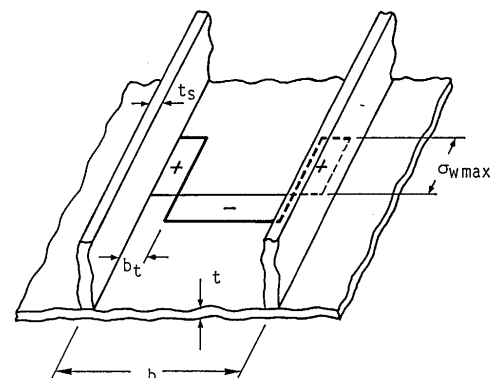
## 2. Research model

A long rectangular plate dealt here is one component plate panel of a longitudinally and transversely stiffened

plate, as shown in Fig.1. Stiffeners are attached to the plate by fillet welding which produces thermal elasto-plastic behavior and consequently inherent strain and inherent deformation in the vicinities of the weld line. A



(a) Long rectangular plate with welding residual stresses and initial deflection



(b) Distribution of residual stresses due to welding of longitudinal stiffeners

Fig.1 Long rectangular plate in study

† Received on Oct.31,1992

\* Professor

\*\* Professor, Hiroshima University

\*\*\* Assistant Professor

\*\*\*\* Engineer, Daikin Industries, Ltd.

rectangular plate with welding residual stress and deformation is subjected to uni-axial compression. Under the compression, each plate panel deforms laterally due to buckling or initial deflection. Lateral deformations of two adjacent panels are supposed opposite. With these discussions, a rectangular plate is assumed to displace uniformly in the plane of the plate and supported simply along the weld line against deflection as illustrated in Fig.1.

3. Welding Residual Stresses

Welding residual stress may be expressed in terms of so-called inherent strain which exists in the vicinity of a weld. The residual stress can be obtained through an elastic analysis using inherent strain as an equivalent load if it is known for the specified materials and welding conditions. According to the recent research [1-4], inherent strain can be predicted easily for several types of joints prepared under the given condition, and so residual stress can be predicted only by the elastic analysis.

In the case of a fillet welded T-joint, such as stiffened plates, the inherent strain distribution is supposed to be different from that in a butt joint, since a part of the given heat input conducts from the plate to the stiffener [1]. However, the height of the stiffener is considerably smaller compared with the width of the plate panel. The temperature history of the stiffener may be considered as uniform and being the same as that of the weld metal, so that the inherent strain and residual stress are approximately the same as those of the weld metal in the plate. For this reason, the distribution of inherent strain, may be regarded as the same as that for a butt joint. With this assumption, the estimated distribution of inherent

strain and the resulting residual stress are a little larger than the accurate one.

3.1 Inherent strain distribution

Recent investigation [1,2] indicates that the inherent strain in the middle portion of a longer rectangular plate surrounded by stiffeners consists only longitudinal components  $\epsilon^*_x$ , which are the same along the longitudinal direction. While near the end of the plate, besides the longitudinal components  $\epsilon^*_x$ , there is also transverse component  $\epsilon^*_y$ , both of which vary little with the change of the width of the plate unless it is too small.

In actual plated structures, such as ships, the length of plate is long enough in comparison with the width so that the important component of residual stress  $\sigma_x$  is caused by  $\epsilon^*_x$ . The inherent strain distribution in butt welded plate of mild steel are obtained by numerical analysis using FEM and shown in Fig.2(a) and its idealization is shown in Fig.2(b). The inherent strain magnitude,  $\hat{\epsilon}^*_x$ , within HAZ and width of inherent strain distribution zone,  $b_0$ , can be expressed as

$$\left. \begin{aligned} b_0 &= 0.242 \xi \alpha EQ / (c \rho t \sigma_Y) \\ \hat{\epsilon}^*_x &= \zeta \sigma_{Yw} / E \end{aligned} \right\} \quad (1)$$

and

$$\left. \begin{aligned} \xi &\doteq 1 - (0.27 \alpha ET_{av}) / \sigma_Y \\ \zeta &\doteq -1 - (0.27 \alpha ET_{av}) / \sigma_Y \\ T_{av} &= Q / c \rho tb \end{aligned} \right\} \quad (2)$$

Where,

- $\alpha$  : linear thermal expansion coefficient,  $1/^\circ\text{C}$
- $c$  : specific heat,  $\text{J/g}^\circ\text{C}$
- $E$  : Young's modulus,  $\text{MPa}$
- $\epsilon^*$  : inherent strain

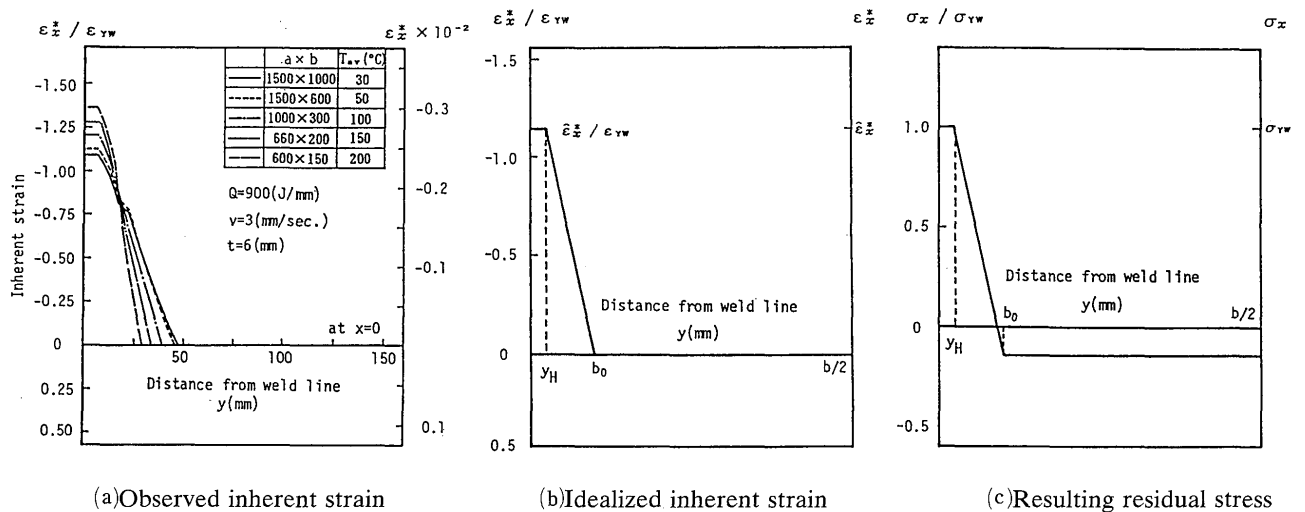


Fig.2 Distribution of inherent strain and residual stress

- t : plate thickness, mm
- k : thermal diffusivity, mm<sup>2</sup>/sec
- b : plate width, mm
- Q : heat input, J/mm
- ρ : density, g/mm<sup>3</sup>
- σ<sub>y</sub> : yield stress of base metal, MPa
- σ<sub>yw</sub> : yield stress of weld metal and HAZ, MPa
- T<sub>av</sub> : average temperature rise, °C

When the welding condition does not change, residual stresses in the butt joints with different aspect ratios a/b can be predicted accurately by the proposed equations [2]. Consequently, accurate residual stress distributions can be obtained.

**3.2 Simple method of prediction of welding residual stresses**

For the middle portion of a longer plate, the longitudinal residual stress due to welding, can be estimated by the following simple method.

The idealized distribution of ε\*<sub>x</sub> is as shown in Fig.2(b), and the resulting residual stress σ<sub>x</sub> for mild steel should be a pattern indicated in Fig.2(c).

The widths y<sub>H</sub> and b<sub>0</sub> of the residual stress distribution coincide with y<sub>H</sub> and b<sub>0</sub> in ε\*<sub>x</sub>-distribution. The maximum tensile stress can be regarded as being equal to the yield stress of the weld metal σ<sub>yw</sub> and the uniform compressive stress can be assumed rationally and its magnitude should be determined so as to satisfy the equilibrium condition of stresses in the cross-section. Then, the stress can be calculated as follows,

$$\sigma_x = E \epsilon_x = \begin{cases} E (\bar{\epsilon}_x - \epsilon^*_x), & 0 \leq y \leq b_0 \\ E \bar{\epsilon}_x & b_0 \leq y \leq b/2 \end{cases} \quad (3)$$

Where,

$$\bar{\epsilon}_x = \frac{(y_H + b_0)}{b - (y_H + b_0)} \hat{\epsilon}^*_x$$

**4. Estimation of welding deformation**

Welding deformation of a rectangular plate due to fillet welding can be expressed in term of inherent deformation as an integration of inherent strain. Here, inherent deformation due to fillet welding and resulting deformation of rectangular plate will be estimated [5].

**4.1 Estimation of angular distortion (inherent deformation) of free T-joint due to fillet welding**

The following equation [6] was proposed for estimation of angular distortion of free T-joint due to single and multi-layers fillet welding, as indicated in Fig.3.

$$\delta_f = (W/W_0)c_1 \chi^m e^{-c_2 \chi} \quad (rad) \quad (4)$$

where, W: wight of total weld metal (per unit weld length), W<sub>0</sub>: weight of weld metal per layer (per unit weld length). When the welding condition is almost the same for each pass and the number of layers is relatively small, (W/W<sub>0</sub>)= n. n: the number of layers, χ = I/(t √vt) × 10<sup>-3</sup>, I: welding current (A), v: welding speed (cm/sec), m, c<sub>1</sub> and c<sub>2</sub>: constants determined by experiments, which are influenced by the kind of steel, welding method, etc. In the case of one pass fillet weld, these coefficients were determined from experiments and they are represented in Table 1.

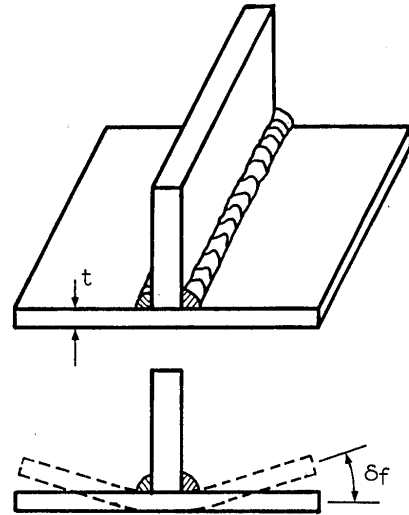


Fig.3 Angular distortion due to fillet weld of T-joint

Table 1 Constants in estimating equation for angular distortion

Material of Plate	σ <sub>y</sub> (kgf/mm <sup>2</sup> )	m	c <sub>1</sub>	c <sub>2</sub>
SM41	29.	1.6	0.36	2.8
SM50	39.	1.9	0.48	2.8
SM53	41.	1.8	0.35	2.5

- Weld Metal : LB-52
- Thickness of Plate t = 6 ~ 40 mm
- Condition of Welding : 300A, 37V, 20.9 cm/min, Q = 31,500 J/cm
- Leg Length : 10 mm

**4.2 Estimation of welding deformation**

Generally welding deformation can be estimated by FEM, imposing the above mentioned inherent deformation δ<sub>f</sub> along the rectangular plate. Here, a simple estimating method is presented.

Welding deflection w<sub>0</sub> of a plate panel is assumed to be expressed by the following equation(5),

$$w_0 = \left( \sum_m A_m \sin \frac{m \pi x}{a} \right) (y^2 - by) \left( -\frac{4}{b^2} \right) \quad (5)$$

where, a: plate length

The coefficient of each term in eqn (5),  $A_m$ , was determined with the aid of the principle of virtual work. The maximum magnitude of the welding deflection,  $w_{0max}$ , was calculated as

$$w_{0max} = \sum_m \left\{ (-1)^{(m-1)/2} \frac{\frac{2b}{\pi m} \delta_{ox} + \frac{2\pi b^4}{15a^3} m \delta_{oy}}{\frac{\pi^4 b^4}{60a^4} m^4 + \frac{\pi^2 b^2}{3a^2} m^2 + 2} \right\} \quad (6)$$

(m takes only odd numbers)

where,  $\delta_{ox} = (1 - (t_s + 2f)/b) \delta_t$ ,  $\delta_{oy} = (1 - (t_s + 2f)/a) \delta_t$ ,  
 $t_s$ : thickness of stiffener, f: leg length of welds

With only six terms, eqn (6) can predict  $w_{0max}$  within an error of 1% with respect to the converged value.

The applicability of eqn (6) was examined in comparison with actual measurement on the deck plate panels of a bulk carrier and car carrier.

Mean values of the maximum deflections of same deck panels are plotted in Fig.4. This figure indicates eqn (6) can predict  $w_{0max}$  very accurately.

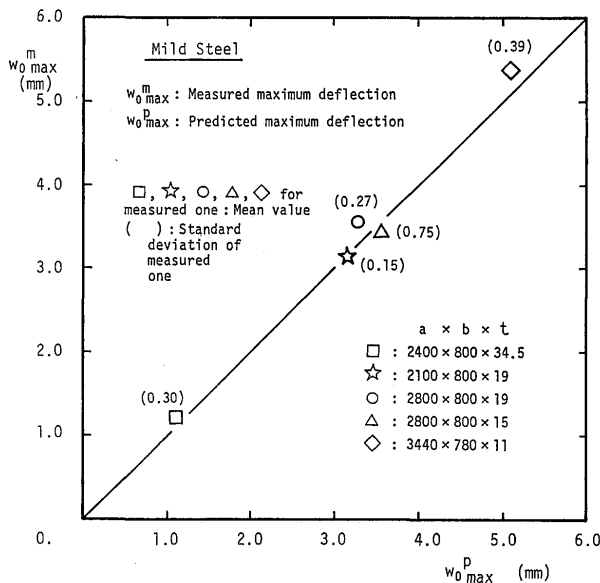


Fig.4 Comparison between predicted maximum initial deflection and measured ones

## 5. Methods for predicting the ultimate strengths of long rectangular plates with uni- and multi-modal initial deflections [7]

### 5.1 Equivalent rectangular plates

In this section, simple methods will be proposed to predict the compressive ultimate strength of a long rectangular plate with multi-modal initial deflections. An equivalent rectangular plate with uni-modal initial deflections is considered, which exhibits the same behaviour as that of the collapsing portion of the rectangular plate with the multi-modal initial deflections.

Taking the same width and thickness as the original plate and denoting the length, width, thickness and the maximum magnitude of initial deflection of this equivalent rectangular plate by  $a_0$ ,  $b$ ,  $t$  and  $A_0$ , respectively, the initial deflection,  $w_0$ , is expressed as

$$w_0 = A_0 \sin \frac{\pi x}{a_0} \sin \frac{\pi y}{b} \quad (7)$$

In the following sections, two different methods will be proposed to determine the length,  $a_0$ , and imperfection magnitude,  $A_0$ , of the equivalent plate; the 'deflection' method for thin plates, and the 'curvature' method for thick plates.

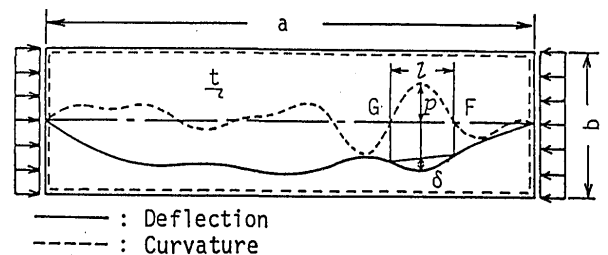


Fig.5 Length of an equivalent plate

### 5.2 Deflection method (for thin plates)

For a thin plate only a particular deflection mode among all component ones becomes predominant and stable above the buckling load. It is postulated that an equivalent plate may show almost the same behaviour as the original if only a component of stable deflection is assumed as an initial deflection. This method is the 'deflection method'.

The following method is proposed to obtain an approximate ultimate strength: (a) The length of the equivalent plate is assumed to be either one half-wave length of the buckling deflection mode, one mode higher,

or two modes higher than the buckling one, with the maximum initial deflection equal to the coefficient of the assumed component of the initial deflection. (b) The ultimate strength is calculated by the elasto-plastic large-deflection analysis or by formulae which will be described in 5.6, on those three equivalent plates. (c) The lowest ultimate strength of the three plates can be regarded as most appropriate.

This method has been applied to twelve panels of the car carrier and three panels of the bulk carrier. The resulting ultimate strengths are compared with the exact ones (as calculated). The predictions are very accurate [7].

**5.3 Curvature method(for thick plates)**

In the case of thick plates, initial plastification takes place just near where the curvature of the initial deflection takes its absolute maximum value. The plastic zone spreads with further load increase, and the plate finally collapses with almost the same deflection mode as the initial deflection. The length,  $a_0$ , and the maximum magnitude of initial deflection,  $A_0$ , of an equivalent plate will be determined by one of the following methods.

**Curvature method I**

For the equivalent plate, the same deflection mode may be used as that of the maximum component of the maximum curvature of the initial deflection, and should indicate the same sign as the maximum curvature. One half-wave length of this deflection component is taken as the length of the equivalent plate. If this component is the  $k$ th, the length,  $a_0$ , should be  $a/k$ . Next, the maximum magnitude of initial deflection,  $A_0$ , is determined so that the maximum curvature of the initial deflection of the equivalent rectangular plate is equal to that of the actual plate considered.

From eqn (7), the maximum curvature of the equivalent plate is  $-A_0 (\pi / a_0)^2$ , for  $x = a_0 / 2$  and  $y = b / 2$ , and this should be equal to  $(1 / \rho_0)_{max}$ . This yields

$$A_0 = - \left( \frac{a_0}{\pi} \right)^2 \left( \frac{1}{\rho} \right)_{max} \quad \text{and} \quad a_0 = \frac{a}{k} \quad (8)$$

**Curvature method II**

There exists two consecutive points where the curvature is zero in the collapsing portion of the plate as illustrated in Fig.5. The point of maximum curvature is located between these. In this method, the distance between these two points,  $\ell$ , is taken as the length of an equivalent rectangular plate. These two points are represented by G and F in Fig.5.

The maximum magnitude of initial deflection,  $A_0$ , is taken as  $-(\ell / \pi)^2 (1 / \rho_0)_{max}$ , so that the maximum magnitude of initial curvature is the same as that of the

original plate. In summary, they are

$$A_0 = - (\ell / \pi)^2 (1 / \rho)_{max} \quad \text{and} \quad a_0 = \ell \quad (9)$$

Applying these two methods, the compressive ultimate strengths of the panels of the bulk carrier are calculated. The results are compared with the accurate values (as calculated). From the comparison, it may be said that both methods predict the ultimate strength fairly well, especially curvature method II.

**5.4 Deflection and curvature methods for medium thickness**

Both the deflection method and curvature method are not very good for plates of medium thickness, but may be applicable. The average of the two results should be an improvement on their individual predictions.

**5.5 Estimation of ultimate strength of equivalent plates using formulae for the minimum strength of plates with a uni-modal initial deflection**

The ultimate strength of an equivalent plate may be calculated by FEM, but a simple method is proposed to use formulae (10) to (17) for the minimum ultimate strength of a plate with a uni-modal initial deflection and welding residual stress to be derived in the next section. For this, the plate parameter,  $\xi = (b / t) \sqrt{(\sigma_w / E)}$ , eqn (16)

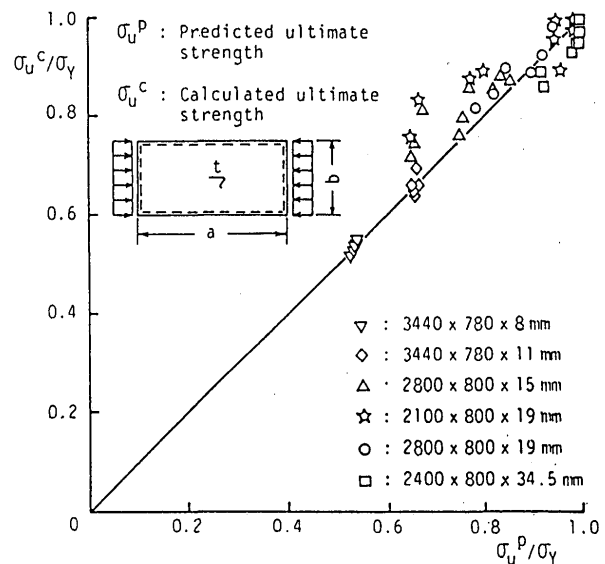


Fig.6 Comparison of calculated compressive ultimate strengths with those predicted by the simple formulae

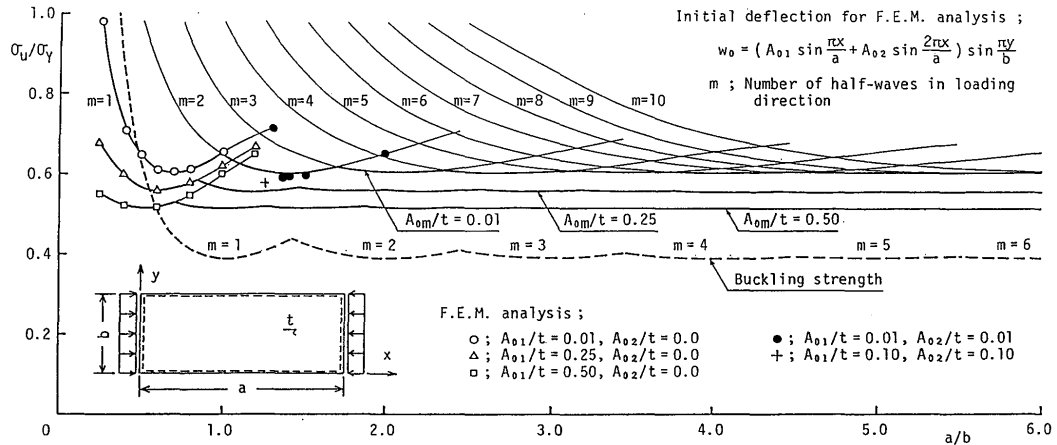


Fig.7 Compressive ultimate strength of rectangular plates with uni-modal initial deflections

, is determined from the plate dimensions. Next,  $\eta = w_0 / t$  is determined according to either the deflection or curvature method. As for welding residual stress, the uniform compressive stress,  $\sigma_c$ , can be determined by the method described in section 3. With these values, the ultimate strength may be calculated.

This method was applied to actual deck plates and the ultimate strengths were compared with those calculated by the elasto-plastic large deflection analysis of FEM. The results are represented in Fig.6 which demonstrates that the proposed prediction method is very accurate.

**5.6 Ultimate strength with a uni-modal initial deflection**

A series of elasto-plastic large deflection analyses have been carried out, assuming that the uni-modal initial deflection is specified and the deflection mode is assumed to be stable until collapse. In Fig.7, the ultimate strength is represented by thin solid lines, and the minimum ultimate strength from those obtained for the same maximum magnitude of uni-modal initial deflection is represented by bold solid lines. The lowest ultimate strength is almost constant for a specified magnitude of initial deflection, irrespective of the aspect ratio.

Taking account of initial deflection and welding residual stresses, the minimum ultimate strengths are calculated as those for  $\sigma_c / \sigma_{yw} = 0.0, 0.11$  and  $0.25$  and those for  $\sigma_c / \sigma_{yw}$  are plotted against the non-dimensional parameter,  $(b/t)\sqrt{(\sigma_{yw}/E)}$  in Fig.8.

The minimum ultimate strengths are expressed by the following simple formulae, which were derived by applying the method of least squares to the calculated values.

(a)  $\sigma_c / \sigma_{yw} = 0.0$

(i)  $0.8 \leq \xi \leq 2.0$   
 $\sigma_u / \sigma_{yw} = (-2.431 \eta^2 + 1.6826 \eta - 0.2961)(\xi^2 - 4.0) + (7.2745 \eta^2 - 4.7431 \eta + 0.6709)(\xi - 2.0) + z_1$  (10)  
 $z_1 = (-0.3597 \eta^2 + 0.1748 \eta + 0.8598) / (2.2432 \eta + 1.3322) + 0.0373 \eta + 0.2481$

(ii)  $2.0 \leq \xi \leq 3.5$   
 $\sigma_u / \sigma_{yw} = (-0.3597 \eta^2 + 0.1748 \eta + 0.8598) / (\xi + 2.2432 \eta - 0.6678) + 0.0373 \eta + 0.2481$  (11)

(b)  $\sigma_c / \sigma_{yw} = 0.11$   
 (i)  $0.8 \leq \xi \leq 1.6$   
 $\sigma_u / \sigma_{yw} = (-0.398 \eta^2 + 0.4339 \eta - 0.1342)(\xi^2 - 2.56) + (1.0814 \eta^2 - 0.7551 \eta + 0.1020)(\xi - 1.6) + z_2$  (12)  
 $z_2 = (0.4974 \eta^2 + 0.8281 \eta + 1.0171) / (2.7942 \eta + 1.2908) - 0.1849 \eta + 0.1571$

(ii)  $1.6 \leq \xi \leq 3.5$   
 $\sigma_u / \sigma_{yw} = (0.4974 \eta^2 + 0.8281 \eta + 1.0171) / (\xi + 2.7942 \eta - 0.3902) - 0.1849 \eta + 0.1571$  (13)

(c)  $\sigma_c / \sigma_{yw} = 0.25$   
 (i)  $0.8 \leq \xi \leq 1.5$   
 $\sigma_u / \sigma_{yw} = (-0.3317 \eta^2 + 0.6314 \eta - 0.2656)(\xi^2 - 2.25) + (0.5369 \eta^2 - 0.7798 \eta + 0.2854)(\xi - 1.5) + z_3$  (14)  
 $z_3 = (0.292 \eta^2 + 1.2936 \eta + 0.7471) / (2.897 \eta + 1.1189) - 0.2715 \eta + 0.2057$

(ii)  $1.5 \leq \xi \leq 3.5$   
 $\sigma_u / \sigma_{yw} = (0.292 \eta^2 + 1.2936 \eta + 0.7471) / (\xi + 2.897 \eta - 0.3811) - 0.2715 \eta + 0.2057$  (15)

where,

$\xi = (b/t)\sqrt{(\sigma_{yw}/E)}$  (16)

$\eta = w_0 / t$  (17)

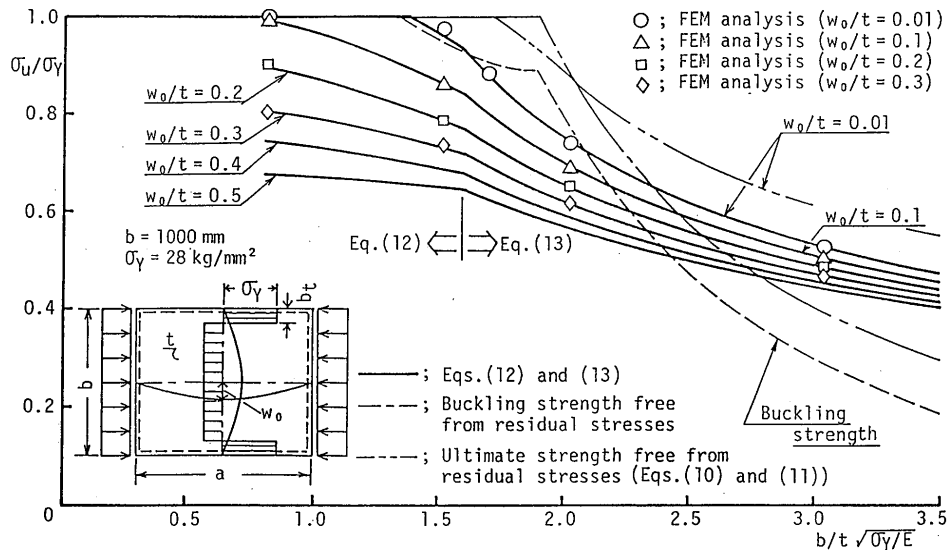


Fig.8 Minimum compressive ultimate strengths of rectangular plates with uni-modal imperfections (Rectangular plates with residual stresses,  $\sigma_c / \sigma_{yw} = 0.11$ )

6. Concluding remarks

Concerning a long plate panel of a longitudinally and transversely stiffened plate, the three estimating methods are presented for (1) welding residual stress, (2) welding deformation and (3) compressive ultimate strength of the plate panel. For these items, basic procedures are described since the space is limited. As further researches have been advanced, details may be reported in individual references (1,2,5,8). Especially, in reference (8), the effective ratio of welding deformation was proposed, which defines the magnitude of the initial deflection of an equivalent plate and can be calculated by simple equations.

Prediction Methods for Welding Deflection and Residual Stress of Stiffened Panels”, Trans. of JWRI, Vol.15, No.2, 1987, pp.197 - 204.

6. Watanabe, M. and Satoh, K., "Control of Angular Distortion Due to Welding Condition in T-fillet Welding Joints - Shrinkage Distortion in Welded Joints (Report 2)", Journal of The Japan Welding Society, Vol.25, No.6, 1956, pp.343 - 348 (in Japanese).
7. Ueda, Y. and Yao, T., "The Influence of Complex Initial Deflection Modes on the Behaviour and Ultimate Strength of Rectangular Plates in Compression", Journal of Constructional steel Research, Vol.5, No.4, 1985, pp.265 - 302..
8. Ueda, Y., Nakacho, K. and Moriyama, S., "Effective Coefficient of Initial Deflection for Simple Prediction Method of Compressive Ultimate Strength of Rectangular Plate", Trans. of JWRI, Vol.17, No.2, 1988, pp.161 - 171.

References

1. Ueda, Y. and Yuan, M.G., "A Predicting Method of Welding Residual Stress Using Source of Residual Stress (Report 3) - Prediction of Welding Residual Stresses in T- and I-joints Using Inherent Strains", Quarterly Journal of The Japan Welding Society, Vol.9, No.3, 1991, pp.19 - 25 (in Japanese).
2. Ueda, Y., Kim, Y.C. and Yuan, M.G., "A Predicting Method of Welding Residual Stress Using Source of Residual Stress (Report 1) - Characteristics of Inherent Strain (Source of Residual Stress)", Trans. of JWRI, Vol.18, No.1, 1989, pp.135 - 141.
3. Ueda, Y. and Yuan, M.G., "A Predicting Method of Welding Residual Stress Using Source of Residual Stress (Report 2) - Determination of Standard Inherent Strain", Trans. of JWRI, Vol.18, No.1, 1989, pp.143 - 150.
4. Ueda, Y. and Yuan, M.G., "Prediction of Residual Stresses in Butt Welded Plates Using Inherent Strains", Submitted to Trans. of ASME, Journal of Engineering Material and Technology.
5. Ueda, Y., Nakacho, K. and Moriyama, S., "Simple