Restraint Stresses and Strains due to Slit Weld in Rectangular Plate (Report I)†

— Formulae for Conventional Restraint Intensities of a Slit in Finite Plate —

Yukio UEDA*, Keiji FUKUDA and You Chul KIM*****

Abstract

Conventional restraint intensities of a slit in a rectangular plate were calculated, under two loading conditions; a uniform dislocation along the slit and uniformly distributed loads along the slit. The calculation was carried out by superposition of the analytical solution for an infinite plate and the correction by the finite element method to a rectangular plate. Furthermore, very accurate formulae for the restraint intensities were derived.

The following conclusions were obtained:

For a specimen having the ratios of $L/l \geq 1.5$ and $B/l \geq 1.8$ (L, B and l are the length, the breadth of the specimen and the length of the slit, respectively.).

(*1*) *The restraint intensities can be easily obtained by the following equations.*

$$R_\delta(x) = [1-\beta_\delta(x)]\frac{E}{\pi}\frac{h}{l}\frac{1}{1-(2x/l)^2} \qquad (\textit{uniform dislocation})$$

$$R_p(x) = (1-\beta_p)\frac{E}{2}\frac{h}{l}\frac{1}{\sqrt{1-(2x/l)^2}} \qquad (\textit{uniformly distributed loads})$$

where $\beta_\delta(x) = [1-(2x/l)^2]\beta_{\delta o}, \quad \beta_{\delta o} = \beta_\delta(x=0)$

$\beta_{\delta o} = 2\beta_p$

$\beta_p = 0.6/(L/l)^n + 0.75/(B/l)^{1.82}, \quad n = 5.8/(B/l)^2 + 2.2$

So, there is a clear relation between the two restraint intensities as shown in the following explicit form;

$$\frac{R_p(x)}{R_\delta(x)} = \frac{\pi}{2}\,\frac{1-\beta_p}{1-\beta_\delta}\,\sqrt{1-(2x/l)^2}$$

(*2*) *Dependencies of the restraint intensities on L/l and B/l are made clear and convergencies of the intensities to limiting values are studied.*

(*3*) *Effect of the ratio of throat thickness to plate thickness upon the restraint intensities is taken into account and a simple formula is derived.*

1. Introduction

There are many metallugical and dynamical factors which influence initiation of cold cracking of weld joints. However, when welding conditions are specified, the initiation of cold cracking may be predicted by the stress and strain induced at a point where the initiation is expected. In this respect, the stress and the strain are the important information to prevent cold cracking from the dynamical point of view.

In one dimensional weld joint, restraint intensity of a weld joint has been used satisfactorily as a representative measure to express the dynamical condition of cold cracking without detailed discussion on the local stress and strain. The main reason exists in a fact that restraint stresses and strains are proportional to a product of the restraint intensity and simple shrinkage of a weld joint.

In contrast with this, when this concept is applied to two dimensional joints such as slit weld, circular—patch weld, etc., there arise several problems: First, restraint stresses and strains are produced by not only simple shrinkage of the welded portion but also thermal deformation of the plate. This deformation influences greatly the resulting stresses and strains. Then, it is rather difficult to predict them simply from the geometric configuration of a joint without considering this thermal deformation. In this connection, the existing definitions of restraint intensity are not rational and the restraint intensities by these difinitions are not alway correlated with the restraint stresses and strains.

In this series of investigation, the following subjects will be studied; (1) the correlation of restraint intensities among the existing definitions, (2) development of accurate formulae for restraint stresses and strains

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* Professor
** Research Associate
*** Graduate Student, Osaka University

and (3) the relation between these restraint intensities and restraint stresses–strains.

In this paper, a slit weld in a finite rectangular plate is considered as a basic two dimensional weld joint (**Fig. 1**). When conventional restraint intensity of a slit weld R is evaluated, two basic loading conditions can be adopted; (a) uniform dislocation along the slit and (b) uniformly distributed loads along the slit. These two loading conditions produce different distributions of restraint intensity along the slit.

First, accurate formulae will be derived for the restraint intensities under these two loading conditions

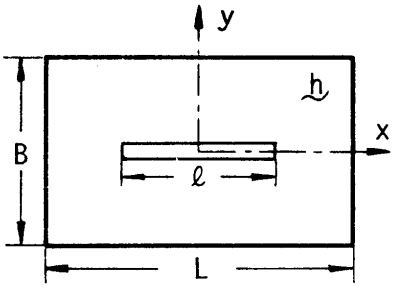


Fig. 1 Slit weld specimen

by superposition of the analytical solution for an infinite plate and that by the finite element method for a rectangular plate. Next, the correlation of the restraint intensities between the two cases will be clarified.

2. Methods of calculation of restraint intensities

When a plate is infinite, the restraint intensities along the slit can be evaluated by adopting the analytical solution. When a plate is finite, a numerical method such as the finite element method is suitable to satisfy the boundary condition of the free edges. However, the finite element method provides numerical values only for a specified size of a finite rectangular plate.

Here, an intention is to develope accurate formulae for restraint intensities of a rectangular plate with arbitrary ratios among the length of a slit, the length and the breadth of the plate, utilizing both advantages of the analytical solution and that by the finite element method.

2.1 Restraint intensity of a slit subjected to a uniform dislocation

When a uniform dislocation in the direction of y axis δ_o is imposed along the slit in an infinite plate, stresses $\{\sigma_e^\delta\}$ at an arbitrary point (x, y) are calculated by the analytical solution[1] as shown below,

$$\left.\begin{aligned}
\sigma_x^\delta &= \frac{E\delta_o}{2\pi}\frac{1}{l}\left[\frac{(1+2x/l)\{(1+2x/l)^2-(2y/l)^2\}}{\{(1+2x/l)^2+(2y/l)^2\}^2} + \frac{(1-2x/l)\{(1-2x/l)^2-(2y/l)^2\}}{\{(1-2x/l)^2+(2y/l)^2\}^2}\right] \\
\sigma_y^\delta &= \frac{E\delta_o}{2\pi}\frac{1}{l}\left[\frac{(1+2x/l)\{(1+2x/l)^2+3(2y/l)^2\}}{\{(1+2x/l)^2+(2y/l)^2\}^2} + \frac{(1-2x/l)\{(1-2x/l)^2+3(2y/l)^2\}}{\{(1-2x/l)^2+(2y/l)^2\}^2}\right] \\
\tau_{xy}^\delta &= \frac{E\delta_o}{2\pi}\frac{1}{l}\left[\frac{\{(1+2x/l)^2-(2y/l)^2\}\,2y/l}{\{(1+2x/l)^2+(2y/l)^2\}^2} - \frac{\{(1-2x/l)^2-(2y/l)^2\}\,2y/l}{\{(1-2x/l)^2+(2y/l)^2\}^2}\right]
\end{aligned}\right\} \quad (1)$$

where E: Young's modulus, l: slit length

From the second equation of Eqs. (1), the stresses along the slit are obtained by substituting $y=0$ as follows,

$$\sigma_s^\infty = \frac{E\delta_o}{2\pi}\frac{1}{l}\left[\frac{1}{1+2x/l} + \frac{1}{1-2x/l}\right] \quad (2)$$

Since Eqs. (1) are for an infinite plate, stresses along the lines $x=\pm L/2$ and $y=\pm B/2$, in the infinite plate can be calculated by Eqs. (1). These stresses must vanish for a finite rectangular plate since the lines under consideration correspond to the free edges.

In order to eliminate these stresses, the finite element method is employed. Changing the sign of the above mentioned stresses, they are applied to the free edges of the rectangular plate under a condition that the displacement in the direction of y axis along the slit should be zero. Then, stresses along the slit line are produced under the load. Superposition of these two stress distributions furnishes the stress distribution for a finite rectangular plate which is subjected to a uniform dislocation along the slit. This procedure is demonstrated in **Fig. 2.**

In the case where a finite rectangular plate is subjected to a uniform dislocation along the slit, the restraint intensity $R_\delta(x)$ is defined as the ratio of the reaction force $\sigma_s^\delta(x)\cdot h$ along the slit to the dislocation δ_s,

$$\begin{aligned} R_\delta(x) &= \sigma_s^\delta h/\delta_s = \sigma_s^\infty h/\delta_o - \sigma_{se}^\delta h/\delta_o \\ &= R_\delta^\infty(x) - \Delta R_\delta = [1-\beta_\delta(x)]R_\delta^\infty(x) \end{aligned} \quad (3)$$

Eq. (3) consists of two terms. The first term indicates the restraint intensity $R_\delta^\infty(x)$ for the infinite plate and the second is a correction term ΔR_δ from the infinite plate to a finite plate. Here, $R_\delta^\infty(x)$ is calculated from Eq. (2)

$$R_\delta^\infty(x) = \frac{E}{\pi}\frac{h}{l}\frac{1}{1-(2x/l)^2} \quad \text{(solution for the infinite plate)} \quad (4)$$

And, $\sigma_{se}^\delta(x)$ and $\beta_\delta(x)$ in Eq. (3) are a correction

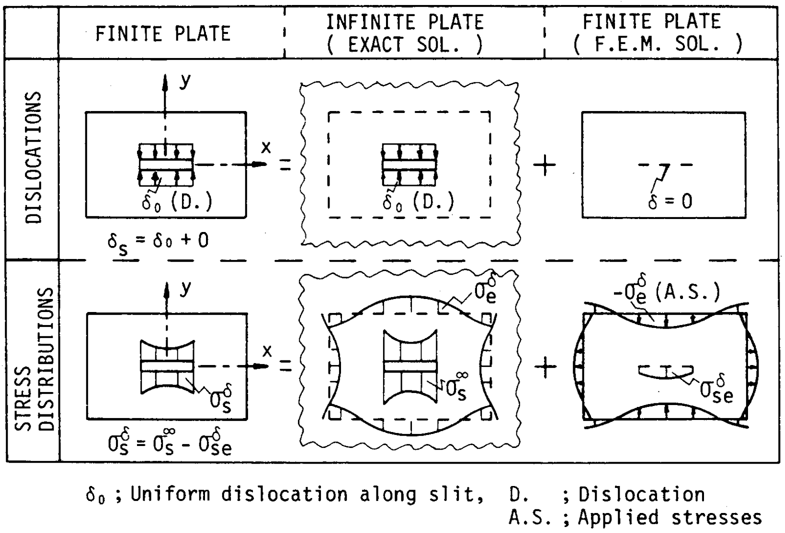


Fig. 2 Procedure for calculation of restraint intensity (uniform dislocation)

stress and a correction factor.

As expected from the principle of Saint–Venant, σ_{se}^{δ} and ΔR_{δ} become uniform along the slit when the size of a rectangular plate becomes greater comparing with the length of the slit. This dynamical condition requires the following ratios, $L/l \geq 1.5$ and $B/l \geq 1.8$.

When the size of a rectangular plate satisfies the above conditions, the correction factor $\beta_{\delta}(x)$ is expressed in the following simple form,

$$\beta_{\delta}(x)=[1-(2x/l)^2]\beta_{\delta o} \tag{5}$$

where $\beta_{\delta o}=\beta_{\delta}(x=0)$

2.2 Restraint intensity of a slit subjected uniformly distributed loads

When uniformly distributed loads in the direction of y axis p_o are imposed along the stit of an infinite plate, stresses $\{\sigma_e^p\}$ at an arbitrary point (x, y) can be calculated from the analytical solution[2] as follows,

$$\left.\begin{aligned}
\sigma_x^p &= \frac{p_o r}{\sqrt{r_1 r_2}}\left[\cos\left(\theta-\frac{\theta_1+\theta_2}{2}\right)\right.\\
&\quad\left.-\frac{(l/2)^2}{r_1 r_2}\sin\theta\sin\frac{3(\theta_1+\theta_2)}{2}\right]-p_o\\
\sigma_y^p &= \frac{p_o r}{\sqrt{r_1 r_2}}\left[\cos\left(\theta-\frac{\theta_1+\theta_2}{2}\right)\right.\\
&\quad\left.+\frac{(l/2)^2}{r_1 r_2}\sin\theta\sin\frac{3(\theta_1+\theta_2)}{2}\right]-p_o\\
\tau_{xy}^p &= \frac{p_o r}{\sqrt{r_1 r_2}}\left[\frac{(l/2)^2}{r_1 r_2}\sin\theta\cos\frac{3(\theta_1+\theta_2)}{2}\right]
\end{aligned}\right\} \tag{6}$$

where $r^2=x^2+y^2$, $r_1^2=(x-l/2)^2+y^2$, $r_2^2=(x+l/2)^2+y^2$
$\tan\theta=y/x$, $\tan\theta_1=y/(x-l/2)$, $\tan\theta_2=y/(x+l/2)$

Stresses along the lines $x=\pm L/2$ and $y=\pm B/2$ can be obtained from Eqs. (6). These stresses must vanish for the finite rectangular plate since the lines under consideration correspond to the free edges. For the infinite plate, the dislocation in the direction of y axis along the slit is obtained in the following form.

$$\delta_s^{\infty}=\frac{2p_o l}{E}\sqrt{1-(2x/l)^2} \tag{7}$$

In order to calculate stresses and deformations of the finite plate under the loading condition, the finite element method is employed to satisfy the boundary conditions along the free edges including the slit. This procedure is illustrated in **Fig. 3.** The resulting dislocation $\delta_s(x)$ along the slit of the rectangular plate is obtained as,

$$\delta_s=\delta_s^{\infty}+\delta_{se} \tag{8}$$

Hence, the restraint intensity of the plate $R_p(x)$ is expressed in the following form.

$$R_p(x)=p_o h/\delta_s=p_o h/(\delta_s^{\infty}+\delta_{se}) \tag{9}$$

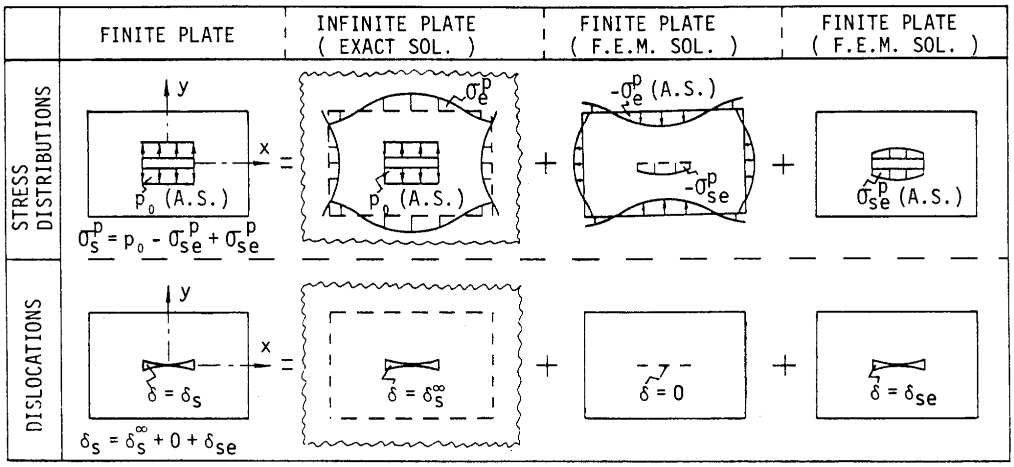
If a rectangular plate without a slit has the ratios of $L/l \geq 1.5$ and $B/l \geq 1.8$, as the same as the previous case, stresses along x axis $-\sigma_{se}^p$ which are produced by stresses along the free edges $-\{\sigma_e^p\}$ are approximately uniform. Then, Eq. (9) may be rewritten as follows,

$$\begin{aligned}R_p(x)&=\sigma_s^p h/\delta_s=p_o h/(\delta_s^{\infty}+\delta_{se})\simeq(p_o-\sigma_{se}^p)h/\delta_s^{\infty}\\&=(1-\sigma_{se}^p/p_o)p_o h/\delta_s^{\infty}=(1-\beta_p)R_p^{\infty}(x)\end{aligned} \tag{10}$$

where $\beta_p=\text{const.}$ (along the slit) (11)

$$R_p^{\infty}(x)=\frac{E}{2}\frac{h}{l}\frac{1}{\sqrt{1-(2x/l)^2}} \quad \text{(solution for the infinite plate)} \tag{12}$$

In the above equation (11), β_p is a correction factor



p_0 ; Uniform stress along slit, A.S. ; Applied stresses

Fig. 3 Procedure for calculation of restraint intensity (uniformly distributed loads)

which is determined by the geometric shape of a rectangular plate with the slit.

2.3 Relation between restraint intensities R_δ and R_p

When a specimen, rectangular plate with the slit, has ratios of $L/l \geq 1.5$ and $B/l \geq 1.8$, the expressions for the restraint intensities R_δ and R_p under the different loading conditions have become simpler. Furthermore, the relation between the correction factors $\beta_{\delta o}$ and β_p is found numerically to be expressed in the following simple form.

$$\beta_{\delta o} \simeq 2\beta_p \tag{13}$$

This relation correlates the restraint intensities R_δ and R_p under the two different loading conditions, as shown in the following explicit form.

$$\frac{R_p(x)}{R_\delta(x)} = \frac{\pi}{2} \frac{(1-\beta_p)\sqrt{1-(2x/l)^2}}{1-2\beta_p[1-(2x/l)^2]} \tag{14}$$

It is seen from the above equation that $R_p(x)$ is easily calculated from $R_\delta(x)$ and vice versa if β_p is expressed in a mathematical form.

Applying the least squares method to the result of calculation for various ratios of L/l and B/l, an expression for β_p is derived as,

$$\beta_p = 0.6/(L/l)^n + 0.75/(B/l)^{1.82} \tag{15}$$

where $n = 5.8/(B/l)^2 + 2.2$

The restraint intensities can be computed from the above equations with an accuracy better than 3% errors.

3. Discussions

3.1 Convergency of restraint intensities of specimens

The restraint intensities at the middle of the slit of a specimen for the two loading conditions are

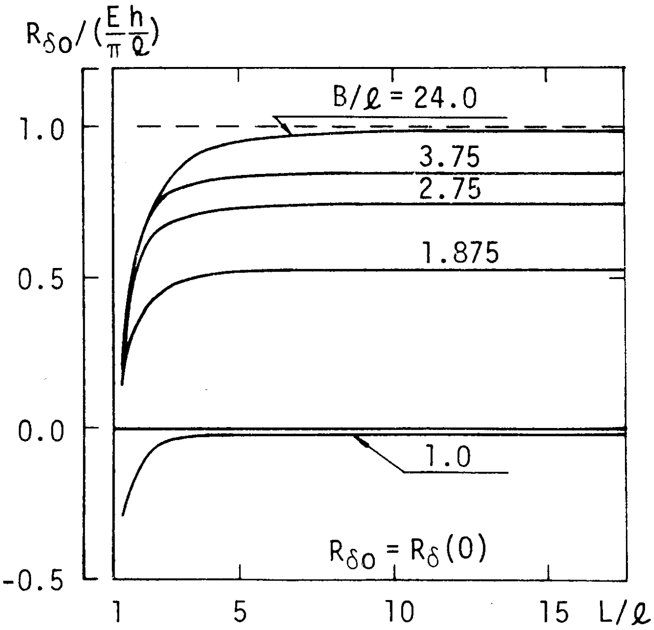


Fig. 4 Restraint intensity at the middle of slit, $R_{\delta o}$ (uniform dislocation)

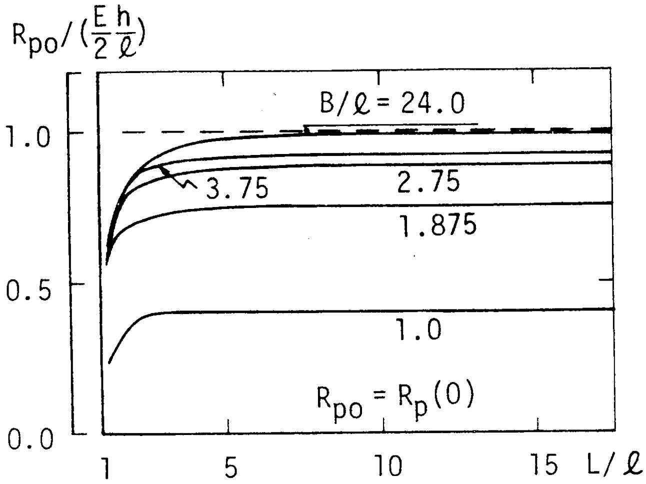


Fig. 5 Restraint intensity at the middle of slit, R_{po} (uniformly distributed loads)

calculated for various ratios of L/l and B/l by the present method. The results are presented in **Figs. 4 and 5.** For a constant value of B/l, the intensities approach to certain values rapidly when the ratio L/l becomes larger. The convergencies of these intensities are dependent upon values of L/l and B/l, and their tendencies are shown in **Figs. 6** and **7.** In the figures, when the intensity becomes 95% of the limiting value, the calculated value is regarded as converged.

In domain I of the figures, the intensity is completely a function of L/l and B/l. In domain II or III, it is influenced only by B/l or L/l. In domain IV, it has reached the limiting value which is the intensity for the infinite plate and it is no longer dependent upon the ratios.

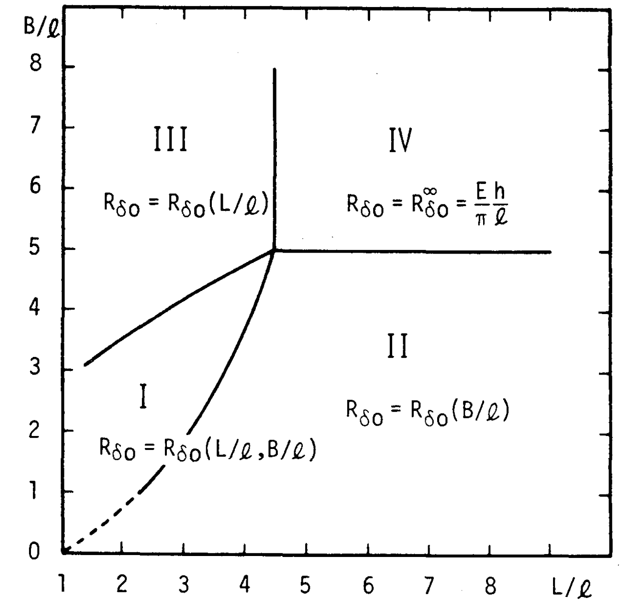


Fig. 6 Effect of parameters L/l and B/l on restraint intensity (uniform dislocation)

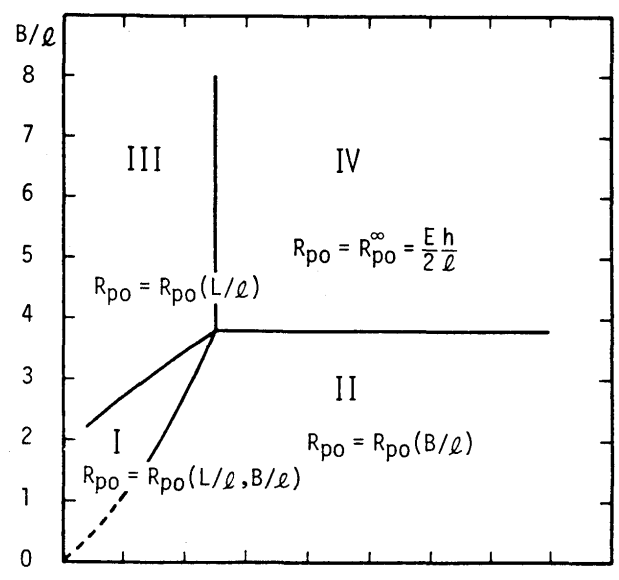


Fig. 7 Effect of parameters L/l and B/l on restraint intensity (uniformly distributed loads)

3.2 Effect of throat thickness on the restraint intensities

When the slit of a thick plate is connected by welding, the ratio of the throat thickness (h_w) to the plate thickness (h) becomes small. The actual restraint intensity is not directly proportional to the thickness of the plate. In one dimensional restraint state of a specimen with any throat thickness, such as the rigid restraint weld cracking test specimen (**Fig. 8**), the

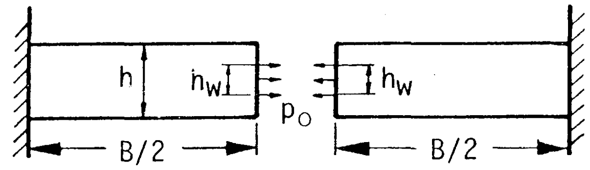


Fig. 8 R.R.C. test specimen

restraint intensity is evaluated analytically with the aid of Fourier series as follows:

$$R=\eta\frac{Eh}{B} \tag{16}$$

where

$$\eta=\frac{1}{1+\sum_{m=1}^{\infty}\frac{8}{m\pi}\left(\frac{\sin m\pi h_w/h}{m\pi h_w/h}\right)^2\frac{\sinh^2 m\pi B/h}{\sinh 2m\pi B/h+2m\pi B/h}} \tag{17}$$

The correction factor η is shown in **Fig. 9.** Then, restraint intensity of the slit of a thick rectangular

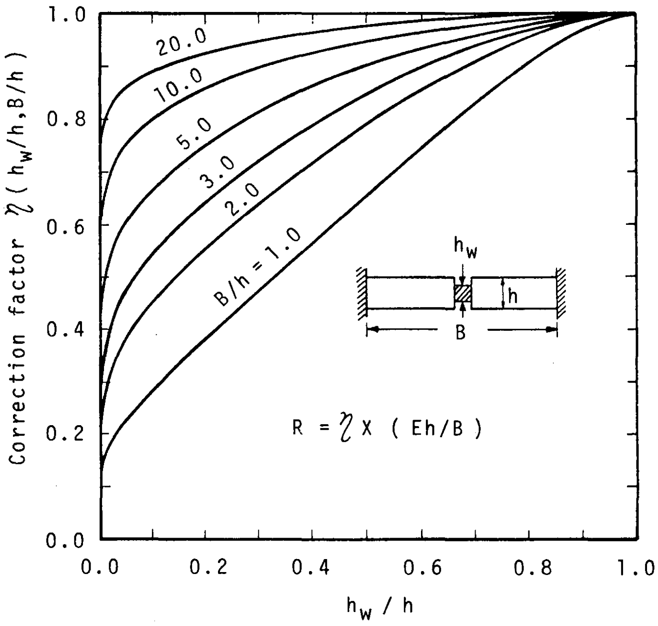


Fig. 9 Relation between ratio of throat thickness to plate thickness and correction factor η for restraint intensity

plate can be easily estimated by Eq. (10) with the aid of correction factor η.

$$R=\eta R_p \tag{18}$$

Applying Eq. (18), the effect of the plate thickness on the restraint intensity is evaluated for the y groove weld cracking specimens (**Fig. 10**).

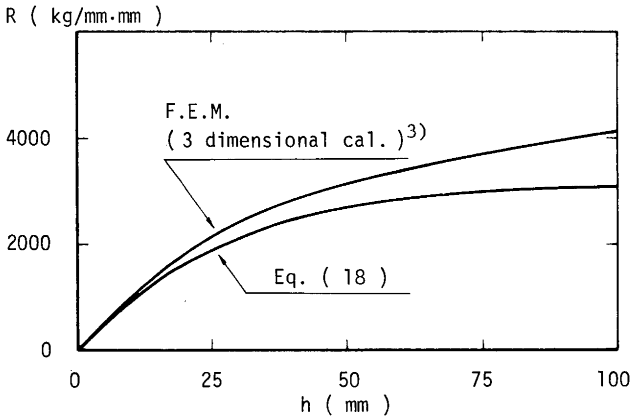


Fig. 10 Effect of plate thickness on restraint intensities at the middle of slit ($L=200$, $B=150$, $l=80$, $h_w=5$ in mm)

4. Conclusions

The restraint intensities of the slit weld specimen under the two loading conditions which are forces producing a uniform dislocation along the slit and uniformly distributed loads along the slit are obtained, by superposition of the analytical solution for the infinite plate and the correction by the finite element method to the finite plate. For the specimens having the ratios of $L/l \geq 1.5$ and $B/l \geq 1.8$, the formulae for the restraint intensities are developed and they are very accurate.

The conclusions obtained in the research are summarized below:

(1) Accurate restraint intensities of the slit weld specimens under the two different loading conditions can be easily calculated.

(2) As the simple relation between the correction factors β_δ and β_p is obtained for specimens with the ratios of $L/l \geq 1.5$ and $B/l \geq 1.8$, the relation between the restraint intensities for the two loading conditions is found and it is expressed in a simple mathematical form.

(3) The convergences of the restraint intensities with respect to the ratios, L/l and B/l are made clear.

(4) In order to take account of the ratio of the throat thickness to the plate thickness into the restraint intensities, a simple correction formula is derived.

References

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